Key

Review: Fractions, Proportion and Ratios

Simplifying Fractions

To simplify a fraction, divide the numerator and denominator by a common factor. Easy common factors to start with are 2 for even numbers and 3 or 5. Keep dividing by common factors until the resulting fraction cannot be divided by any other common factor and is in lowest terms.

Example:

Simplify $\frac{18}{27}$ ← *numerator* / ← *denominator*

Solution A: Simplify, using a factor of 9

$$\frac{18 \div 9}{27 \div 9} = \boxed{\frac{2}{3}}$$

Solution B: Simplify, using a factor of 3, twice

$$\frac{18 \div 3}{27 \div 3} = \frac{6 \div 3}{9 \div 3} = \boxed{\frac{2}{3}}$$

Practice 1: Simplify these fractions to their lowest terms. Show your work!

a. $\frac{4 \div 2}{16 \div 2} = \frac{2 \div 2}{8 \div 2} = \boxed{\frac{1}{4}}$

or $\frac{4 \div 4}{16 \div 4} = \boxed{\frac{1}{4}}$

b. $\frac{3 \div 3}{12 \div 3} = \boxed{\frac{1}{4}}$

c. $\frac{25 \div 25}{75 \div 25} = \boxed{\frac{1}{3}}$

d. $\frac{15 \div 3}{21 \div 3} = \boxed{\frac{5}{7}}$

e. $\frac{8 \div 2}{18 \div 2} = \boxed{\frac{4}{9}}$

f. $\frac{45 \div 5}{100 \div 5} = \boxed{\frac{9}{20}}$

g. $\frac{20 \div 10}{50 \div 10} = \boxed{\frac{2}{5}}$

h. $\frac{3 \div 3}{21 \div 3} = \boxed{\frac{1}{7}}$

i. $\frac{7 \div 7}{56 \div 7} = \boxed{\frac{1}{8}}$

Mixed and Improper Fractions

Improper fractions occur where the numerator is larger than the denominator. For example: $\frac{5}{3}$

Improper fractions may be expressed as mixed numbers containing a whole number and proper fraction. For example: $1\frac{2}{3}$

$\frac{5}{3} = 1\frac{2}{3}$ ← equivalent fractions

Convert mixed to improper fraction: Multiply the whole number and denominator. Add the product to the numerator.

Example:

$3\frac{1}{2}$ multiply → $3 \times 2 = 6$ then add $6 + 1 = 7$ ⇒ $\boxed{\frac{7}{2}}$

Practice 2: Convert to improper fractions. Show your work!

a. $2\frac{3}{4}$ $2 \times 4 + 3 = 11$ $\boxed{\frac{11}{4}}$

b. $1\frac{5}{8}$ $1 \times 8 + 5 = 13$ $\boxed{\frac{13}{8}}$

c. $3\frac{4}{5}$ $3 \times 5 + 4 = 19$ $\boxed{\frac{19}{5}}$

d. $10\frac{2}{3}$ $10 \times 3 + 2 = 32$ $\boxed{\frac{32}{3}}$

e. $2\frac{2}{15}$ $2 \times 15 + 2 = 32$ $\boxed{\frac{32}{15}}$

f. $5\frac{3}{5}$ $5 \times 5 + 3 = 28$ $\boxed{\frac{28}{5}}$

Convert improper to mixed fraction: Divide the numerator by the denominator to find the whole number. This is how many times the bottom number can go into the top number. The amount left over (remainder) will stay in the fraction as the numerator.

Examples:

$\frac{12}{5}$ $12 \div 5 = 2$ remainder 2 ($2 \times 5 = 10$, $12 - 10 = 2$) $\boxed{2\frac{2}{5}}$

$\frac{14}{3}$ $14 \div 3 = 4$ remainder 2 $\boxed{4\frac{2}{3}}$

Practice 3: Convert to mixed fractions. Show your work!

a. $\frac{8}{3}$ $8 \div 3 = 2 \text{ r}2$ $\boxed{2\frac{2}{3}}$

b. $\frac{27}{5}$ $27 \div 5 = 5 \text{ r } 2$ $\boxed{5\frac{2}{5}}$

c. $\frac{15}{4}$ $15 \div 4 = 3 \text{ r}3$ $\boxed{3\frac{3}{4}}$

d. $\frac{135}{10}$ $135 \div 10 = 13 \text{ r}5$ $\boxed{13\frac{5}{10}}$ → $13\frac{1}{2}$ reduce

Proportional Reasoning

A ratio is a comparison between two numbers measured in the same units.
A ratio can be expressed in three ways as shown below:

- in words by using the word "to" 9 to 16
- as a fraction $\frac{9}{16}$
- a notation using colon : 9 : 16

Ratios, like fractions, can be simplified. For example, the ratio 150 : 15 can also be expressed: $\frac{150}{15}$

which can be simplified: $\frac{150 \div 15}{15 \div 15} = \frac{10}{1}$

Notice that the numerator of the fraction is larger than the denominator. This can be common with ratios.

If two ratios are equivalent (equal), the first (top) term of each ratio compares to the second (bottom) term in an identical manner. You can represent this equivalence in the two ratios here:

$$\frac{150}{15} = \frac{10}{1}$$

An equation showing equivalent ratios is called a proportion.

Cross Multiply and Divide

When two fractions are equal to each other, any unknown numerator or denominator can be found. The following example shows the process.

Example: Find x when $\frac{x}{3} = \frac{2.1}{4}$

Solution: Cross multiply means multiply the numbers across the equals sign (the arrow). The divide part means divide that result by the number opposite the unknown (x) as shown below.

$\frac{x}{3} = \frac{2.1}{4}$ this give the result $x = 3 \times 2.1 \div 4$

It does not matter where the unknown (x) is in the proportion, This process works for all situations.

Example: Find x when $\frac{4}{x} = \frac{10}{3}$

divide

cross multiply

$x = 4 \times 3 \div 10$

$x = 12 \div 10$

$\boxed{x = 1.2}$

This process can also be used when one side of the equal sign is not in fraction form.

Example: Find x when $27 = \frac{x}{3}$

The whole number 27 can be represented as a fraction $\frac{27}{1}$ → $\frac{27}{1} = \frac{x}{3}$ $27 \times 3 \div 1$

$x = 81 \div 1$

$\boxed{x = 81}$

Practice 4: Find the missing term by using cross multiply and divide. If necessary, round answers to one decimal place. Show your work!

a. $\frac{6}{x} = \frac{8}{2}$ $6 \times 2 \div 8$ $\boxed{x = 1.5}$

b. $\frac{x}{5} = \frac{20}{4}$ $5 \times 20 \div 4$ $\boxed{x = 25}$

c. $\frac{3}{x} = \frac{2}{4}$ $3 \times 4 \div 2$ $\boxed{x = 6}$

d. $\frac{3}{15} = \frac{x}{6}$ $3 \times 6 \div 15$ $\boxed{x = 1.2}$

e. $\frac{7}{x} = \frac{30}{1}$ $7 \times 1 \div 30$ $\boxed{x = 0.2}$

f. $\frac{x}{5} = \frac{12}{1}$ $5 \times 12 \div 1$ $\boxed{x = 60}$

Working with Ratios

Ratios are often used in problems to express the relationship between parts.

Example: Charlie works as a cook in a restaurant. His chicken soup recipe contains:

- 11 cups seasoned broth
- 5 cups diced vegetables
- 3 cups rice
- 3 cups chopped chicken

Write the ratio for each of the following relationships.

a) vegetables to chicken 5 : 3

b) broth to vegetables 11 : 5

c) chicken to rice 3 : 3 → 1 : 1 reduce

d) chicken to the total ingredients in the recipe

total 11 + 5 + 3 + 3 = 22

3 : 22

Practice 5:

1. A conveyor belt has 2 pulleys. One pulley has a diameter of 45 cm and the other has a diameter of 20 cm. What is the ratio of the smaller diameter to the larger diameter?

20 : 45 → reduce by ÷ 5 $\boxed{4:9}$

small : large

2. On a bicycle with more than one gear, the ratio between the number of teeth on the front gear and the number of teeth on the back gear determines how easy it is to pedal. If the front gear has 30 teeth and the back gear has 10 teeth, what is the ratio of front teeth to back teeth?

30 : 10 → reduce by ÷ 10 $\boxed{3:1}$

front : back.

3. What is the ratio of 250 mL of grape juice concentrate to 1 L of water? (Hint: 1000 mL = 1L)

↳ 1000 mL

250 : 1000 → reduce by ÷ 10 $\boxed{25:100}$

* Ratios must be in same units!

Working with Proportion

When given a ratio and one of the parts, write a proportion to solve using cross multiply and divide. Use a letter or word to represent the parts to put the numbers in the correct location.

Example: For a painting, Greg mixes inks to get the tint he wants. He uses a ratio of yellow ink to white ink of 3:1 and red ink to yellow in of 2:3.

a) How many mL of yellow ink would he use if he used 500 mL of white ink?

$\frac{\text{yellow}}{\text{white}}$ $\frac{3}{1} = \frac{x}{500}$ $3 \times 500 \div 1$

$\boxed{x = 1500 \text{ mL yellow ink}}$

b) How many mL of red ink would he need if he used 750 mL of yellow ink?

$\frac{\text{red}}{\text{yellow}}$ $\frac{2}{3} = \frac{x}{750}$ $2 \times 750 \div 3$

$\boxed{x = 500 \text{ mL red ink}}$

Practice 6:

1. If a secretary types 55 words in one minute, how long will it take the secretary to type a 2000 word report?

$\frac{\text{words}}{\text{time}}$ $\frac{55}{1} = \frac{2000}{x}$ $2000 \times 1 \div 55 = 36.36$

$\boxed{x = 36.4 \text{ min}}$

2. The ratio between Siu's height and Tai's height is 5:6. If Tai is 145 cm tall, how tall is Siu, to the nearest whole centimetre?

$\frac{\text{Siu}}{\text{Tai}}$ $\frac{5}{6} = \frac{x}{145}$ $5 \times 145 \div 6$

$\boxed{x = 120.8}$

3. A mechanic can rotate the 4 tires on a truck in 15 minutes. How many minutes would it take the mechanic to rotate the tires on 5 trucks? Hint: what are you comparing??

↳ 20 tires

$\frac{\text{tires}}{\text{time}}$ $\frac{4}{15} = \frac{20}{x}$ $20 \times 15 \div 4$

$300 \div 4$ $\boxed{x = 75 \text{ min}}$

More Working with Proportion

Sometimes, the information given in a proportion question requires you to take a different approach to solving. This occurs when a part of the whole is being found.

Example: Tom and Susan made $180 from their garage sale. Tom contributed fewer items do the money was divided between Tom and Susan in a ratio of 1:2. How much money did each person receive?

Solution: Since the ratio is 1:2, this means the money is divided into 3 parts of which Tom get one and Susan gets 2. When solving this problem, one proportion for each person is needed, with the comparison to the total money earned.

$\frac{\text{part}}{\text{whole}}$

Susan

$\frac{2}{3} = \frac{x}{180}$

$2 \times 180 \div 3$

$360 \div 3$

$\boxed{x = 120}$

Tom

$\frac{1}{3} = \frac{x}{180}$

$1 \times 180 \div 3$

$180 \div 3$

$\boxed{x = 60}$

these should add to total.

Practice 7: More Working with Proportion

$2+1=3$

1) The ratio of flour to butter in a recipe for pie crust is 2:1. If a baker makes 30 cups of piecrust, how many cups of flour and how many cups of butter does he need?

$\frac{\text{part}}{\text{Whole}}$

Flour

$\frac{2}{3} = \frac{x}{30}$

$x=20$

butter

$\frac{1}{3} = \frac{x}{30}$

$x=10$

2) A compound of two chemicals is mixed in a ratio of 3:12. If there are 45 L of the compound, how much of each chemical is in the mixture?

$3+12=15$

$\frac{\text{part}}{\text{Whole}}$

$\frac{3}{15} = \frac{x}{45}$

$x=9$

$\frac{12}{15} = \frac{x}{45}$

$x=36$