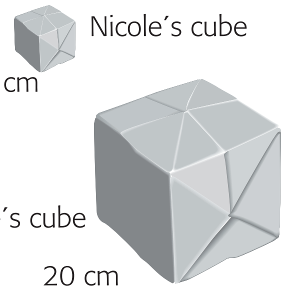
2.5 Combining Powers

YOU WILL NEED

- a calculator



GOAL

Simplify products and quotients of powers with the same exponent.

LEARN ABOUT the Math

Nicole and Yvonne made origami paper cubes for a math project.

? How will the volume and surface area of Yvonne's cube compare to those for Nicole's cube?

EXAMPLE 1 | Comparing the surface area and volume of cubes

Nicole's Solution

$$\begin{aligned} \text{Surface area} &= 6 \text{ faces} \times \text{area of one face} \\ &= 6 \times 4^2 \\ &= 6 \times 4 \times 4 \quad (\text{or } 6 \times 16) \\ &= 96 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Volume} &= \text{length} \times \text{width} \times \text{height} \\ &= 4^3 \\ &= 64 \text{ cm}^3 \end{aligned}$$

I calculated the surface area and volume of my cube.

$$\begin{aligned} \text{Surface area} &= 6 \times (4 \times 5)^2 \\ &= 6 \times (4 \times 5) \times (4 \times 5) \\ &= 6 \times (4 \times 4) \times (5 \times 5) \\ &= 6 \times 4^2 \times 5^2 \\ &= 6 \times 16 \times 25 \\ &= 2400 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Volume} &= (4 \times 5)^3 \\ &= (4 \times 5) \times (4 \times 5) \times (4 \times 5) \\ &= (4 \times 4 \times 4) \times (5 \times 5 \times 5) \\ &= 4^3 \times 5^3 \\ &= 64 \times 125 \\ &= 8000 \text{ cm}^3 \end{aligned}$$

I calculated the surface area and volume of Yvonne's cube. I wrote the side length of 20 as 4×5 to make it easier to compare to my cube.

$$\frac{2400}{96} = \frac{25}{1}$$

The surface area of Yvonne's cube is 25 times greater than that of my cube.

I wrote the ratio of the surface area of Yvonne's cube to the surface area of my cube, and then simplified.

$$\frac{8000}{64} = \frac{125}{1}$$

The volume of Yvonne's cube is 125 times greater than mine.

I wrote the ratio of the volume of Yvonne's cube to the volume of my cube, and then simplified.

Reflecting

A. How could Nicole have predicted she could calculate the surface area of Yvonne's cube by multiplying her own cube's surface area by 25?

B. How could Nicole have predicted that she could calculate the volume of Yvonne's cube by multiplying her own cube's volume by 125?

C. How do Nicole's calculations show why $(4 \times 5)^2 = 4^2 \times 5^2$ and $(4 \times 5)^3 = 4^3 \times 5^3$?

WORK WITH the Math

EXAMPLE 2 | Simplifying the base of a power

Yvonne calculated the volume of a cube with a side length of 7 cm as 343 cm^3. How can she use that calculation to figure out the volume of a cube with a side length of 14 cm?

Yvonne's Solution

$343 = 7^3$

The volume of the new cube is 14^3.

I knew that $14 = 2 \times 7$, so I could use the exponent law or I could write $(2 \times 7)^3 = (2 \times 7) \times (2 \times 7) \times (2 \times 7)$
That's the same as $2 \times 2 \times 2 \times 7 \times 7 \times 7$.
I realized that I could just multiply the old volume of 7^3 by 2^3.
That's an easy multiplication.

$$\begin{aligned} 14 &= 2 \times 7 \\ 14^3 &= (2 \times 7)^3 \\ &= 2^3 \times 7^3 \\ &= 8 \times 7^3 \end{aligned}$$

The volume of a cube with a side length of 14 cm is $8 \times 343 = 2744$ cm^3.

EXAMPLE 3 | Evaluating powers with different bases

Evaluate $2^5 \times 5^4$.

Shelby's Solution

$$
\begin{aligned}
&2^5 \times 5^4 \\
&\quad = 2 \times 2 \times 2 \times 2 \times 2 \\
&\qquad \times 5 \times 5 \times 5 \times 5
\end{aligned}
$$

I wrote the expression using repeated multiplication.

$$
\begin{aligned}
&\quad = 2 \times (2 \times 5) \times (2 \times 5) \\
&\qquad \times (2 \times 5) \times (2 \times 5)
\end{aligned}
$$

I rearranged the 2s and 5s because $2 \times 5 = 10$, and that's easier to multiply by than 2s or 5s.

$$= 2 \times 10 \times 10 \times 10 \times 10$$

I multiplied the 2s by the 5s.

$$
\begin{aligned}
&= 2 \times 10^4 \\
&= 2 \times 10\ 000 \\
&= 20\ 000
\end{aligned}
$$

I simplified using powers.

EXAMPLE 4 | Simplifying expressions involving powers

Simplify $(2^3 \times 4^2)^3$.

Austin's Solution

$$(2^3 \times 4^2)^3 = (2^3 \times 2^4)^3$$

I noticed that 4^2 can be expressed as a power with a base of 2, where $4^2 = (2^2)^2$ or 2^4.

$$
\begin{aligned}
&= (2^{3+4})^3 \\
&= (2^7)^3
\end{aligned}
$$

I simplified using the product law.

$$= 2^{21}$$

I could simplify even further using the power of a power law.

EXAMPLE 5 | Simplifying powers in fraction form

Simplify $\left(\frac{-3^2}{4^3}\right)^3$.

Derek's Solution

$$\left(\frac{-3^2}{4^3}\right)^3 = \frac{(-3^2)}{(4^3)} \times \frac{(-3^2)}{(4^3)} \times \frac{(-3^2)}{(4^3)}$$
$$= \frac{-3^2 \times -3^2 \times -3^2}{4^3 \times 4^3 \times 4^3}$$
$$= \frac{(-3^2)^3}{(4^3)^3}$$

I figured out what the expression meant by using repeated multiplication and the rules for multiplying fractions.

I realized that I could have just applied the power to the numerator and denominator separately.

$$= \frac{-3^{2\times3}}{4^{3\times3}}$$
$$= \frac{-3^6}{4^9}$$

I simplified using the exponent law for a power of a power.

In Summary

Key Idea

- An exponent can be applied to each term in a product or quotient involving powers.

  That is, $(ab)^m = a^m b^m$ and $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ $(b \neq 0)$.

  For example, $(3 \times 7)^2 = 3^2 \times 7^2$ and $\left(\frac{3}{7}\right)^2 = \frac{3^2}{7^2}$.

Need to Know

- Sometimes an expression is easier to evaluate if you simplify it first; for example, $2^5 \times 5^5$ is easier to evaluate when it is simplified to $(2 \times 5)^5 = 10^5$ and $2^3 \times 8^2$ is easier to evaluate if you rewrite it as a single power of 8: $2^3 \times 8^2 = 8^1 \times 8^2 = 8^3$.

Checking

1. Express as a product or quotient of two powers.

a) $(2 \times 3)^4$ **b)** $\left(\frac{2}{3}\right)^5$ **c)** $(3^2 \times 5^4)^3$ **d)** $\left(\frac{3^3}{7^2}\right)^2$

2. Write each expression as a power with a single base. Show your work.

a) 2×4 **b)** $(3^2 \times 9)^3$ **c)** $(4^2 \times 16^2)^4$ **d)** $\left(\frac{5^2}{5}\right)^4$

Practising

3. Write each expression as a power with a single base. Show your work.
   a) $(3 \times 7)^2$ b) $(4 \times 6)^3$ c) $(9 \div 3)^2$ d) $(24 \div 3)^3$

4. Simplify. Express as a single power where possible.
   a) $(8^3 \times 5^2)^4$
   b) $(4^3 \times 3^2)^2(4^5 \times 3^2)^3$
   c) $[(2^4)(3^3)]^2(2^2 \times 3^3)^3$
   d) $\left(\frac{4^6}{4^4}\right)^3$
   e) $\left(\frac{2^4}{7^2}\right)^3$
   f) $\frac{(2^5 \times 5^2)^2}{(2^4 \times 5)^2}$

5. Evaluate.
   a) $(2^3 \times 3^2)^2$
   b) $(3^2 \times 1^2)^2(3^2 \times 1^2)^3$
   c) $\left(\frac{5^5}{5^3}\right)^3$
   d) $\frac{(2^6 \times 4^3)^2}{(2^3 \times 4^2)^2}$

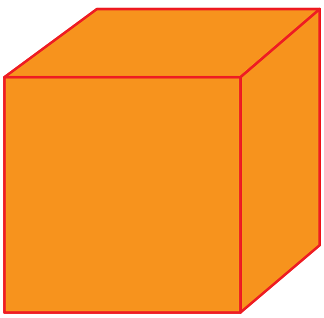
6. **Multiple choice.** Simplify $(2^2 \times 4^2)^3$.
   A. 2^{18} B. 2^{24} C. 4^{18} D. 2^9

7. **Multiple choice.** Simplify $(1.8^3 \times 1.8^2)^2$.
   A. 1.8^7 B. 1.8^{12} C. 1.8^6 D. 1.8^{10}

8. **Multiple choice.** Simplify $\left(\frac{5^6}{5^2}\right)^4$.
   A. 5^{16} B. 5^{12} C. 5^{24} D. 5^5

9. Kalyna can only enter one-digit numbers on her calculator. The exponent key and the display are working fine. Explain how she can evaluate each power using her calculator.
   a) 25^4 b) 16^2

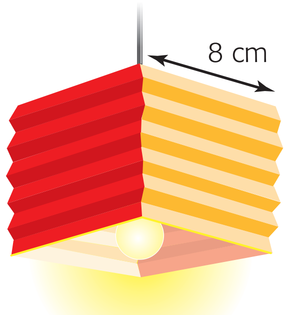
10. Simplify $4^3 \times 250^3$, to make it easier to evaluate. Show your work.



11. The side length of a cube is 3^5 units.
    a) Determine the surface area of the cube without using powers.
    b) Determine the surface area using powers.
    c) Did you prefer the method you used in part a) or part b)? Explain why.
    d) Determine the volume without using powers.
    e) Determine the volume using powers.
    f) Did you prefer the method you used in part d) or part e)? Explain why.

12. Navtej wants to paint her room and is on a budget. She found a 4 L can of paint, in a colour that she liked, on the mistints shelf at the hardware store. She knows that 500 mL covers 6 m^2. She wants to use two coats of paint. Represent the area that she is able to paint using a power. Recall that 1 L = 1000 mL.

13. Hye-Won is making ornamental paper lanterns for her Chinese New Year party. Her first lantern is a cube.



a) Express the volume of the lantern as a power.

b) Another lantern has a volume of 2^{15} cm^3. How many times as high is that cube than the first lantern?

14. Describe two different ways to evaluate $\frac{6^3}{2^3}$. Which would you use? Why?

15. Suppose you are asked to evaluate $2^8 \times 25^4$ and $10^5 \times 8^3$. Which expression might you simplify first? Which one might you not simplify? Explain.

Reading Strategy

Evaluating

Find someone who used a different way from you in questions 14 and 15. Justify your choices to each other.

Closing

16. Explain how can you simplify $40^3 \times 5^5$ to calculate it using mental math.

Extending

17. a) Can you express $(0.81)^3$ as an equivalent power with a single base of 0.9, $(0.81)^3 = 0.9^{■}$? Explain how you know.

b) Can you express $(0.9)^3$ as an equivalent power with a base of 0.81, $(0.9)^3 = (0.81)^{■}$? Explain how you know.

c) When can you express a power with a base of 0.9 as an equivalent power with the base of 0.81?

18. Express each amount as a power with a single base. Show your work.

a) $(0.25^4 \times 0.5^2)^3$ b) $(1.2^3 \times 1.44)^2$ c) $\left(\frac{0.16^3}{0.4^3}\right)^3$