# 2.4 Multiplying and Dividing Powers

**YOU WILL NEED**

- a calculator

**GOAL**

**Simplify products and quotients of powers with the same base.**

## *INVESTIGATE the Math*

Derek wants to determine the value of this expression: $(5^4)(5^4) \div (5^2)^3$. He wonders if he can write it so that it will be easier to calculate the value.

**? How can Derek simplify this expression?**

**A.** Rewrite Derek's expression as a fraction, with powers in both the numerator and denominator.

**B.** Write the numerator using repeated multiplication.

**C.** Express the numerator as a single power. How does this new power relate to the original powers in the numerator?

**D.** Write the denominator using repeated multiplication.

**E.** Express the denominator as a single power. How does this new power relate to the original powers in the denominator?

**F.** Write the quotient as a single power. How does this new power relate to the other two?

**G.** How could Derek have simplified his original expression?

## Reflecting

**H.** Why does it make sense that sometimes you add, sometimes you subtract, and sometimes you multiply exponents to simplify expressions involving powers?

**I.** In each case, write a rule you can use to simplify
- the product of two powers with the same base
- the quotient of one power and another with the same base
- a power raised to an exponent

**J.** Why do the bases need to be the same for some of the exponent rules you wrote in part I to work?

## *WORK WITH the Math*

### EXAMPLE 1 | Simplifying numerical expressions using exponent laws

Simplify. **a)** $(3^2)(3^4)$ **b)** $6^5 \div 6^3$ **c)** $(4^2)^5$

**Bay's Solution**

**a)** $(3^2)(3^4) = 3^{2+4}$
$= 3^6$

The powers are to be multiplied, and their bases are the same.
I added the exponents.

**b)** $6^5 \div 6^3 = 6^{5-3}$
$= 6^2$

The powers are to be divided, and their bases are the same.
I subtracted the exponents.

**c)** $(4^2)^5 = 4^{2\times 5}$
$= 4^{10}$

The power is to be raised to an exponent.
I multiplied the exponents.

### EXAMPLE 2 | Simplifying algebraic expressions using exponent laws

Simplify. **a)** $(x^6)(x^5)$ **b)** $x^7 \div x^2$ **c)** $(x^5)^4$

**Derek's Solution**

**a)** $(x^6)(x^5) = x^{6+5}$
$= x^{11}$

The powers are to be multiplied, and their bases are the same.
I added the exponents.

**b)** $x^7 \div x^2 = x^{7-2}$
$= x^5$

The powers are to be divided, and the bases are the same.
I subtracted the exponents.

**c)** $(x^5)^4 = x^{5\times 4}$
$= x^{20}$

The power is to be raised to an exponent.
I multiplied the exponents.

### EXAMPLE 3 | Simplifying using several exponent laws

Simplify. **a)** $(-2)^7(-2)^3 \div [(-2)^2]^3$ **b)** $\dfrac{(y^3)^5}{(y)(y^4)}$

**Shelby's Solution**

$$(-2)^7(-2)^3 \div [(-2)^2]^3 = (-2)^{7+3} \div [(-2)^2]^3$$
$$= (-2)^{10} \div [(-2)^2]^3$$

I added the exponents of the powers that were multiplied.

$= (-2)^{10} \div (-2)^{2\times 3}$
$= (-2)^{10} \div (-2)^{6}$

I multiplied the exponents of the power in the divisor.

$= (-2)^{10-6}$
$= (-2)^{4}$

I subtracted the exponent of the divisor.

$(-2)^7(-2)^3 \div [(-2)^2]^3 = (-2)^4$ or 16

**b)** $\frac{(y^3)^5}{(y)(y^4)} = \frac{y^{3\times 5}}{y^{1+4}}$

I multiplied the exponents of the power in the numerator and added the exponents of the powers in the denominator.

$= \frac{y^{15}}{y^{5}}$

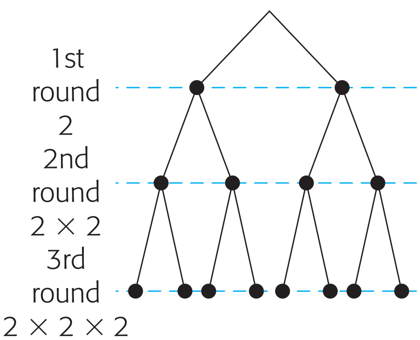
$= y^{15-5}$
$= y^{10}$

I subtracted the exponent of the divisor.

## EXAMPLE 4 | Representing a power as an equivalent power

Austin and Shelby want to spread the news about school picture day. Austin will call two people and ask each one to call two more people, and so on. Shelby will call four people and ask each one to call four more people, and so on. Shelby says, with her plan, the same number of people would be called on the fourth round of calls as on the eighth round of calls with Austin's plan. Is Shelby right?

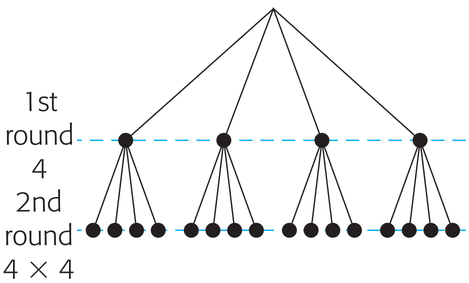
### Austin's Solution: Representing $2^8$ as a power with a base of 4




I drew a diagram of my plan. The number of people called doubles with each round. So, $2^1$ people will be called in round 1, $2^2$ people in round 2, and $2^3$ people in round 3.

In round 8, $2^8$ people will be called using my plan.

To represent the number of calls in round 8 as a power, I think the base should be 2 and the exponent should be the number of the round.




I drew a diagram of the first two rounds of Shelby's plan. The number of people called is multiplied by 4 with each round. So, $4^1$ people will be called in round 1, $4^2$ people in round 2, and so on.

In round 4, $4^4$ people will be called using Shelby's plan.

$$2^8 = (2 \times 2) \times (2 \times 2) \times (2 \times 2) \times (2 \times 2)$$
$$= (2^2)(2^2)(2^2)(2^2)$$
$$= (2^2)^4$$
$$= 4^4$$

Shelby is right.

To compare the number of people called under my plan to Shelby's plan, I paired the 2s and wrote each pair as $2^2$, which is equal to 4. I knew I had four 4s multiplied together. After round 8, $4^4$ people would be called under my plan.

## In Summary

### Key Ideas

- **Exponent law for products**
  To simplify the product of two powers with the same base, keep the base the same and add the exponents.

  $(a^m)(a^n) = a^{m+n}$; for example, $(2^2)(2^3) = 2^{2+3} = 2^5$.

- **Exponent law for quotients**
  To simplify the quotient of two powers with the same base, keep the base the same and subtract the exponents.

  $(a^m) \div (a^n) = a^{m-n} (a \neq 0)$; for example, $(2^5) \div (2^3) = 2^{5-3} = 2^2$.

- **Exponent law for a power of a power**
  To raise a power to an exponent, keep the base the same and multiply the exponents. $(a^m)^n = a^{mn}$; for example, $(4^2)^3 = 4^{2 \times 3} = 4^6$.

### Need to Know

- The exponent laws only work when the powers have the same base; for example, you can't multiply $(3^2)(5^2)$ using the exponent law for powers.

## Checking

**1.** Simplify.

**a)** $(9^2)(9^7)$

**b)** $7^2 \times 7^5 \times 7$

**c)** $\frac{2^6}{2^5}$

**d)** $\frac{(5^4)^2}{5^8}$

**e)** $\frac{(11^8)^5}{11^9}$

**f)** $(8^8)(8^3) \div (8^2)^2(8^2)$

**2.** Express each number as a power with a different base.

**a)** 16

**b)** $4^3$

**c)** $9^4$

**3.** Simplify.

**a)** $(x^4)(x^6)$

**b)** $a^8 \div a^6$

**c)** $(m^3)^4$

## Practising

**4.** Express each as a power with a single exponent.

a) $(10^6)(10^7)$
b) $(3^4)^2(3^3)$
c) $\frac{12^5}{12^2}$
d) $(2^3)^3 \div 2^4$
e) $(6^3)^5 \div (6^2)^4$
f) $\frac{(5^7)(5^5)}{(5^4)^2(5^2)}$

**5.** Evaluate.

a) $(-2)^3(-2)^2$
b) $(-1)^4(-1)^7$
c) $\frac{(-11)^7}{(-11)^5}$
d) $\frac{[(-8)^2]^3}{(-8)^5}$
e) $\frac{[(-3^{10})]^2}{[(-3)^8]^2}$
f) $\frac{(-6)^9(-6)^9}{[(-6)^3]^3(-6)^3}$

**6.** Determine the exponent that makes each statement true.

a) $2^6 = 4^{\blacksquare}$
b) $6^6 = 216^{\blacksquare}$
c) $625^2 = 25^{\blacksquare}$
d) $27^4 = 3^{\blacksquare}$

**7.** **Multiple choice.** Which is not equivalent to $(3^3)(3^4)$?

**A.** $3^{12}$
**B.** 2187
**C.** $3^7$
**D.** $(3^3)(3^2)(3^2)$

**8.** **Multiple choice.** For which exponent is $2^4 = 4^{\blacksquare}$ true?

**A.** 1
**B.** 2
**C.** 3
**D.** 4

**9.** **Multiple choice.** For which exponent is $4^3 = 2^{\blacksquare}$ true?

**A.** 3
**B.** 4
**C.** 5
**D.** 6

**10.** Use a numerical example to illustrate each exponent law.

a) $(a^m)(a^n) = a^{m+n}$
b) $a^m \div a^n = a^{m-n} \ (a \neq 0)$
c) $(a^m)^n = a^{mn}$

**11.** Oksana solved the following question:

$$\frac{2^3 \times 2^8}{(2^3)(2^3)^2} = \frac{2^{24}}{(2^3)(2^6)}$$
$$= \frac{2^{24}}{2^9}$$
$$= 2^{15}$$
$$= 32\,768$$

When she checked the answer with her calculator she got 4. Identify the mistake Oksana made.

**12.** Express each as a power with a single exponent.

a) $(x^3)(x^2)$
b) $\frac{y^7}{y^2}$
c) $(s^2)^3(s^5)$
d) $\frac{(p^5)^3}{p^{11}}$

**13.** Determine if each solution is correct or incorrect. If a solution is incorrect, correct the error and solve.

**a)** $\frac{(3^2)^3(3^4)}{3^7}$
$= \frac{(3^6)(3^4)}{3^7}$
$= \frac{3^{10}}{3^7}$
$= 3^3$
$= 27$

**b)** $\frac{4^8 \times 4^2}{(4^4)(4^2)^3}$
$= \frac{4^{10}}{(4^4)(4^5)}$
$= \frac{4^{10}}{4^9}$
$= 4^1$
$= 4$

**c)** $\frac{[(-7)^5]^3(-7)^5}{[(-7)^3]^3(-7)^7}$
$= \frac{(-7)^{15}(-7)^5}{(-7)^9(-7)^7}$
$= \frac{(-7)^{20}}{(-7)^{15}}$
$= (-7)^5$
$= -16\,807$

**14.** **a)** Simplify $\frac{3^5}{3^5}$ by first writing the powers as products.

**b)** Simplify $\frac{3^5}{3^5}$ using the exponent law for quotients.

**c)** Evaluate $\frac{3^5}{3^5}$.

**d)** How does knowing the exponent laws for quotients help explain why $a^0 = 1$?

**e)** Discuss whether $a^0$ would have a similar meaning for any value of $a$ (except 0).

**15.** How do you know that $10^{\square} \neq 8^{\square}$ if the two powers are whole numbers?

**16.** Write each power in a simplified form.

**a)** $4^6$ as a power of 2

**b)** $27^5$ as a power of 3

**c)** $9^6$ as a power of $(-3)$

**d)** $(-125)^8$ as a power of $(-5)$

**17.** Simplify.

**a)** $(x^4)(x^2)^2$

**b)** $\frac{(m^5)^2}{m^8}$

**c)** $[(y)(y^2)]^3$

**d)** $(a^2)^2$

**e)** $(a^2)(a^2)(a^2)$

**f)** $\frac{(b)(b^5)(b^4)}{b^5}$

## Closing

**18.** Explain why $3^5 \times 3^4 = 3^9$, but $3^5 \times 4^3 \neq 12^8$.

## Extending

**19.** **a)** Is there a whole number for which $3^{20} = 4^{\square}$? Explain why or why or not.

**b)** Can you write $5^4 \times 125^3$ as a single power? Explain why or why or not.

**c)** Can you write $5^x + 5^y$ as a power of 5? Explain why or why or not.