# 2.2 Expressing a Number as a Power

**GOAL**

**Use powers to represent repeated multiplication.**

**YOU WILL NEED**

- a calculator

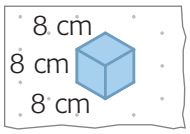

## *LEARN ABOUT* the Math

Yvonne uses square sticky notes to leave messages for her mom. She decides to make a cube-shaped holder for the notes in woodworking class. She wants the holder to hold eight packs of notes. Each sticky-note pack is a cube and each sticky note is 8 cm wide.

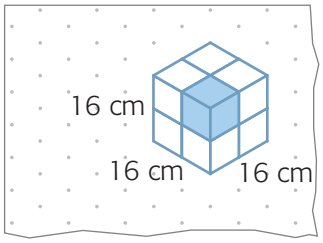
**What should the capacity of the container be?**

**EXAMPLE 1** Representing volume using a power

### Yvonne's Solution




Each sticky-note pack is a perfect cube; each pack has dimensions 8 cm by 8 cm by 8 cm.




I want the holder to be in the shape of a cube and I want it to hold eight packs. I must make the large cube two packs wide, two packs high, and two packs long. The larger cube is 16 cm by 16 cm by 16 cm.

The capacity is $16 \times 16 \times 16 = 4096 \text{ cm}^3$.
The capacity of the container is $16^3 \text{ cm}^3$.

I described the capacity of my holder with a power. The side length is the base of the power and the number of dimensions is the exponent.

## Reflecting

**A.** How can Yvonne use the fact that $4^3 = 64$ to calculate $16^3$?

**B.** Why can you use powers to describe $4 \times 4 \times 4$, but not to describe $2 \times 3 \times 4$?

## *WORK WITH the Math*

**EXAMPLE 2** | Evaluating a power

Evaluate $\left(-\frac{1}{2}\right)^2$ and $-\left(\frac{1}{2}\right)^2$.

**Derek's Solution**

$\left(-\frac{1}{2}\right)^2 = \left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right)$

I wrote the power as a repeated multiplication. I had to repeat everything inside the brackets.

$-\left(\frac{1}{2}\right)^2 = \frac{1}{4}$

I multiplied from left to right.

$-\left(\frac{1}{2}\right)^2 = (-1)\left(\frac{1}{2}\right)^2$
$= (-1) \times \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right)$
$= \left(-\frac{1}{2}\right) \times \left(\frac{1}{2}\right)$
$-\left(\frac{1}{2}\right)^2 = -\frac{1}{4}$

I wrote the power as a repeated multiplication. I didn't repeat the minus sign with each $\frac{1}{2}$, because it wasn't in brackets.

I multiplied from left to right.

The two powers represent different numbers.

**Communication *Tip***

A base without an exponent is understood to have an exponent of 1; so, $5 = 5^1$.

**EXAMPLE 3** | Evaluating a power by using a pattern

Evaluate $3^0$, $5^0$, $(-1)^0$, and $0^0$.

**Nicole's Solution**

$3^3 = 3 \times 3 \times 3 = 27$
$3^2 = 3 \times 3 = 9 \quad (27 \div 3)$
$3^1 = 3 \quad (9 \div 3)$
$3^0 = 1 \quad (3 \div 3)$
I expect that $3^0 = 3^1 \div 3 = 1$.

I used a pattern. I started with an exponent of 3. I decreased the exponent by 1. I noticed that when the exponent decreases by 1, the value of the power is divided by 3.

$5^3 = 5 \times 5 \times 5 = 125$
$5^2 = 5 \times 5 = 25 \quad (125 \div 5)$
$5^1 = 5 \quad (25 \div 5)$
$5^0 = 1 \quad (5 \div 5)$

I did the same with $5^0$. The result was the same.

$(-1)^3 = (-1)(-1)(-1) = -1$
$(-1)^2 = (-1)(-1) = 1 \quad (-1 \div -1)$
$(-1)^1 = -1 \quad (1 \div -1)$
$(-1)^0 = 1 \quad (-1 \div -1)$

I got the same result with $(-1)^0$, although in this case, the value of the power just flipped between $-1$ and 1.

$0^3 = 0 \times 0 \times 0 = 0$
$0^2 = 0 \times 0 = ? \quad (0 \div 0)$
$0^0$ is undefined.

I tried the same pattern with $0^0$ but it didn't work this time. I can't write these expressions as the previous value divided by 0, because division by 0 is undefined, not 0. I can't use the pattern to determine the value of $0^0$.

## In Summary

### Key Idea

- Powers are used to represent repeated multiplication. The base represents the number being multiplied and the exponent, when it is a whole number, tells how many times the base appears. For example, $7^6 = 7 \times 7 \times 7 \times 7 \times 7 \times 7$ and $\left(\frac{6}{7}\right)^3 = \left(\frac{6}{7}\right)\left(\frac{6}{7}\right)\left(\frac{6}{7}\right)$

### Need to Know

- Any power with a nonzero base and an exponent of 0 is equal to 1; that is, $x^0 = 1$, $x \neq 0$.
- If there are no brackets in a power, the exponent applies only to its positive base: $-3^4 = (-1)(3 \times 3 \times 3 \times 3) = -81$.
  $-3^4$ is the opposite of $3^4$, just as $-3$ is the opposite of 3.
- A power has a negative base when the base is negative and is enclosed in brackets. For example, $(-3)^4 = (-3)(-3)(-3)(-3) = 81$.

## Checking

**1.** Represent each repeated multiplication as a power.

**a)** $5 \times 5 \times 5 \times 5 \times 5 \times 5$

**b)** $(3.2 \times 3.2) \times (3.2 \times 3.2)$

**c)** $(-4)(-4)(-4)$

**d)** $-(7)(7)(7)(7)(7)$

**e)** $\left(\frac{5}{7}\right)\left(\frac{5}{7}\right)\left(\frac{5}{7}\right)$

**f)** $\left(\frac{3}{4}\right)\left(\frac{3}{4}\right)\left(\frac{3}{4}\right)\left(\frac{3}{4}\right)\left(\frac{3}{4}\right)$

## Practising

**2.** Represent each repeated multiplication as a power.

**a)** $4 \times 4 \times 4 \times 4 \times 4 \times 4$

**b)** $(6 \times 6) \times (6 \times 6)$

**c)** $(-5.4)(-5.4)(-5.4)$

**d)** $-(8)(8)(8)(8)(8)$

**e)** $\left(\frac{8}{9}\right)\left(\frac{8}{9}\right)\left(\frac{8}{9}\right)\left(\frac{8}{9}\right)$

**f)** $\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)$

**3.** Represent each power using repeated multiplication.

**a)** $2^4$ **b)** $(-2)^4$ **c)** $-2^4$ **d)** $-(-2)^4$

**4.** Evaluate each power.

**a)** $-7^3$ **b)** $(-7)^3$ **c)** $(-3)^4$ **d)** $-3^4$ **e)** $-12.4^2$ **f)** $(-12)^2$

**5.** Complete the table.

| | Power | Base | Exponent | Repeated Multiplication | Value in Standard Form |
|---|---|---|---|---|---|
| **a)** | $9^4$ | | | | 6561 |
| **b)** | | 5 | | $(5)(5)(5)$ | |
| **c)** | | $-2$ | 5 | | |
| **d)** | | | | $-(6)(6)(6)$ | |
| **e)** | $-4^6$ | | | | |

**6. Multiple choice.** Which power does not represent 256?

**A.** $2^8$ **B.** $4^4$ **C.** $8^3$ **D.** $16^2$

**7. Multiple choice.** Which statement is true?

**A.** $3.1^3 = 3.1 \times 3.1 \times 3.1$ **C.** $-3^3 = (3)(3)(3)$

**B.** $(-1)^6 = -1$ **D.** $-6^2 = 36$

**8. Multiple choice.** Evaluate $(-5)^4$.

**A.** $-625$ **B.** $25$ **C.** $625$ **D.** $-54$

**9.** Shelby says that for any power with a positive integer base, when the base and exponent are switched, the greater power is always the one with the greater base. Do you agree or disagree? Justify your decision.

**10.** If a power has a negative integer base, can you predict whether the power has a positive or negative value? Explain.

**11.** Arrange in order from least to greatest.

$$-2^4,\ (-2)^4,\ -(-2^2),\ (-1)^{100},\ (-1)^{31}$$

**12.** Ihor read that, in Japan, some farmers grow watermelons inside cubes so the melons grow in the shape of a cube. He bought a sheet of special plastic that is 45.0 cm by 70.0 cm.

**a)** Determine the area of the sheet of plastic.

**b)** The surface area of a cube is $6s^2$, where $s$ is the length of one side. Determine the dimensions of the side length of the largest plastic cube Ihor can build.

**13.** **a)** Calculate $2^4$, $3^4$, $4^4$, and $5^4$.
**b)** Calculate $2^5$, $3^5$, $4^5$, and $5^5$.
**c)** The fourth power of one number is 13 greater than the fifth power of another. What are the numbers?
**d)** How could you have predicted that the bases in part c) would be fairly small?

**14.** **a)** Complete the pattern: $1^1 = 1$, $1^2 = 1 \times 1 =$ ■, $1^3 =$ ■, $1^4 =$ ■, $1^5 =$ ■
**b)** Use the pattern to evaluate $1^x$, where $x$ is any whole number.

**15.** **a)** Order the following from least to greatest: $6^4$, $6^3$, $6^2$, $6^0$, $6^1$.
**b)** Would the order change if you replaced 6 with 5, $-5$, or 0?

**16.** Order the following from least to greatest: $2^3$, $3^2$, $3^4$, $4^3$, $3^5$, $5^3$.

**17.** Represent each repeated multiplication as a power.
**a)** $s \times s \times s \times s$
**b)** $(-y)(-y)$
**c)** $(t \times t) \times (t \times t)$
**d)** $-(p)(p)(p)$

**18.** Marilyn has 49 pennies, 32 nickels, 9 dimes, 25 quarters, 8 loonies, and 16 toonies in a jar.
**a)** Write a power to represent the number of each type of coin.
**b)** Write an equation using the powers in part a) to represent the total number of coins.

## Closing

**19.** Derek says $4^5 > 5^4$, since a power with a higher exponent is always greater. Do you agree? Explain.

## Extending

**20.** **a)** Evaluate $(-2)^3(-2)^4$, $(-2)^2(-2)^6$, and $(-2)(-2)^5$.
**b)** Express each answer in part a) as a power with a base of $(-2)$.
**c)** Look for a pattern. How could you get the power in part b) just by looking at the question in part a)?

**21.** Sue wanted to invite all 128 families of the Grade 9 class at her school to the Math Olympics, an evening of math games and contests. She didn't have time to call every family herself, so she decided to call two families and ask each person she called to call two more families, and so on.
**a)** Determine how many rounds of calls will be needed.
**b)** Represent the number of families as a power.