2.1 Modelling Squares and Cubes

YOU WILL NEED

- a calculator
- centimetre cubes
- centimetre grid paper

power

a numerical expression that shows repeated multiplication; e.g., the power 2^3 is a shorter way of writing $2 \times 2 \times 2$. It is read as "two to the third" or "two cubed"—2 is the **base** and 3 is the **exponent**. We say 2 has the exponent 3.

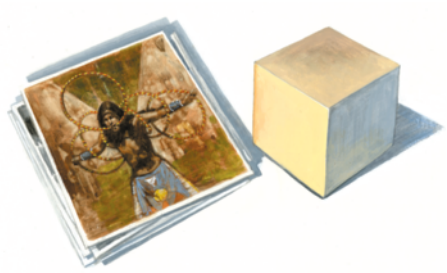
2^3 ← exponent

base →

GOAL

Represent perfect squares and perfect cubes using models.

INVESTIGATE the Math



Yvonne is making a gift for her sister's Naming Ceremony. It will be a cube with a square photo on each face. The sides of the photos will be natural number centimetre lengths. She wants the cube to be as large as possible, but she is mailing it and it cannot be more than 5000 cm^3 in volume.

? What should be the dimensions of the cube?

A. Complete the second and third rows in the table expressing each side length, face area, and volume as a **power**.

Side Length of Cube, s (cm)	Side Length as a Power, s^1	Area of Face of Cube, $s \times s$ (cm^2)	Area of Face as a Power, s^2	Volume of Cube, $s \times s \times s$ (cm^3)	Volume of Cube as a Power, s^3
2	2^1	$2 \times 2 = 4$	2^2	$2 \times 2 \times 2 = 8$	2^3
4				$4 \times 4 \times 4 = 64$	4^3
8		$8 \times 8 = 64$			

base

the number used as a factor in a power

exponent

the number used to express the number of factors in a power

B. Continue to complete rows for other side lengths as necessary.

C. What should be the side length of Yvonne's cube? Explain how you know.

Reflecting

D. You can represent the value 64 with two different models, s^2 and s^3. How is the model of the form s^2 different from the model of the form s^3?

E. How do you know that 225 can represent the area of one of the square faces of a cube with natural number centimetre lengths, but not the volume?

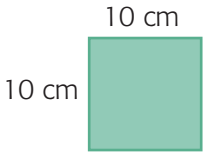
F. How do you know that 343 can represent the volume of a cube like Yvonne's, but not the area of one face?

WORK WITH the Math

EXAMPLE 1 | Modelling square powers

A square wall tile has an area of 100 cm^2. Represent the area of this tile as a geometric model and as a power.

Bay's Solution



The tile is square, so the geometric model must be square too. Since $100 = 10 \times 10$, each side of the square must be 10 cm.

$100 = 10^2$

100 is a **perfect square**, so I wrote it as a power using a base of 10 and an exponent of 2.

perfect square
the product of a natural number multiplied by itself; e.g., 49 is a perfect square because $7 \times 7 = 49$.

EXAMPLE 2 | Modelling cube powers

A softball comes in a cube-shaped box with a volume of 1728 cm^3. Represent the volume of this box as a geometric model and as a power.

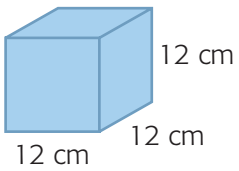
Amanda's Solution

The box is a cube.

Each side of the box must have the same length and the base must be a square. I knew the cube would be more than 10 cm on a side, because 1728 is more than $10^3 = 1000$.

$12 \times 12 \times 12 = 1728$

I tried 12. Since the volume is even, I knew the side length must also be even.



The geometric model is a cube that looks like this.

$1728 = 12^3$

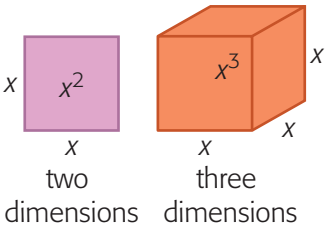
1728 is a **perfect cube**, so I wrote it as a power using a base of 12 and an exponent of 3.

perfect cube
the product of a natural number multiplied by itself twice; e.g., 343 is a perfect cube because $7 \times 7 \times 7 = 343$.

In Summary

Key Idea

- You can represent some powers using a geometric model. For example, you can represent a perfect square as the area of a square with natural-number-length sides and a perfect cube as the volume of a cube with natural-number-length sides.



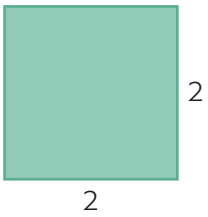
Need to Know

- A perfect square can be written as a power: $n^2 = n \times n$.
- A perfect cube can be written as a power: $n^3 = n \times n \times n$.

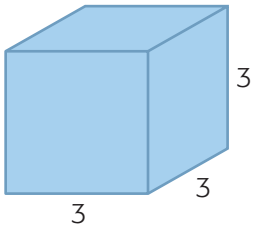
Checking

1. Represent each geometric model as a power.

a)



b)



2. a) Write $6 \times 6 \times 6$ as a power. **b)** Write 11×11 as a power.

3. a) Determine the side length of a square with an area of 81 m^2.
b) Determine the side length of a cube with a volume of 8 cm^3.

Practising

4. Determine the value of ■.

a) $4 \times 4 = ■^2$ **c)** $■^2 = 100$ **e)** $7^2 = ■$

b) $3 \times 3 \times 3 = ■^3$ **d)** $■^3 = 27$ **f)** $5^3 = ■$

5. A square floor mat has a side length of 5 m. Write the area of the mat as a power.

6. The side length of a cube is 12 cm. Determine the following:
a) the area of one face **b)** the **surface area** **c)** the volume

7. Joga is making palak paneer. He used a large cube of cheese that had a volume of 3375 cm^3.
a) Sketch a model of the cheese. Label the side lengths.
b) Joga sliced the cheese into 3 cm cubes. How many cubes did he have?

8. How many more perfect squares than perfect cubes are there between 1 and 1000?

9. **Multiple choice.** A square floor mat has a side length of 22 m. What is the area of the mat as a power?
   A. $222\ m^2$ **B.** $22^3\ m^3$ **C.** $22^2\ m^2$ **D.** $2^{22}\ m^2$

10. **Multiple choice.** Determine the area of one face of a cube with a side length of 14 cm.
    A. $196\ cm^2$ **B.** $196\ cm^3$ **C.** $14\ cm^2$ **D.** $2744\ cm^3$

11. **Multiple choice.** Determine the volume of a cube with a side length of 14 cm.
    A. $196\ cm^2$ **B.** $196\ cm^3$ **C.** $14\ cm^2$ **D.** $2744\ cm^3$

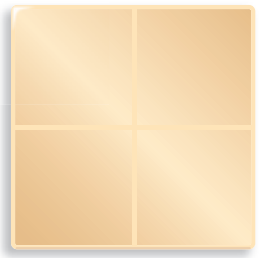
12. Sketch geometric models for 4^2 and 4^3. How are the models alike and different?

13. Austin says that he can draw a geometric model for any power of 2. Do you agree or disagree with him? Justify your decision.

14. Two perfect squares have a difference of 169.
    a) How far apart are the square roots?
    b) How far apart are the cubes of the values in part a)?

15. Which numbers have the same values as their square and their cube?

16. Nasri is creating a mosaic using tiles for art class. He has a frame that is 60 cm by 60 cm and divided into four sections. The frame's border is 2 cm wide. He has many tiles with these dimensions: 1 cm by 1 cm, 2 cm by 2 cm, 3 cm by 3 cm, 5 cm by 5 cm, and 10 cm by 10 cm. Sketch some designs for Nasri's mosaic. Use graph paper to help you.



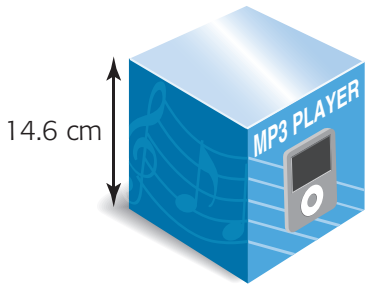
Closing

17. How could you prove to someone that there are more perfect squares than perfect cubes in the numbers between 100 and 200?

Extending

18. Nicole and her friend Hélène are preparing *sucre à la crème*. They use plates that are 20 cm by 30 cm and cut the treats into 2 cm cubes. They will sell 10 cubes for $1.00. They hope to raise about $50. How many plates will Nicole and Hélène need?

19. Sean and Damien bought Patrick an MP3 player for his birthday. They have a sheet of wrapping paper that is 30 cm by 60 cm. Can they wrap the box without cutting the paper? Sketch how you know.



20. You have seen that 64 is a perfect square and a perfect cube. Determine two other numbers with this property.