

$$\begin{aligned} \text{Area of outer cylinder} &= \pi(1.0)^2 + 2\pi(1.0)(0.6) \\ &\doteq 6.6 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{Area of inner cylinder} &= \pi(0.8)^2 + 2\pi(0.8)(0.6) \\ &\doteq 5.0 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{Area of top edge} &= \text{area of large circle} - \text{area of small circle} \\ &= \pi(1.0)^2 - \pi(0.8)^2 \\ &\doteq 1.0 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} SA &= \text{area of outer cylinder} \\ &\quad + \text{area of inner cylinder} + \text{area of top edge} \\ &\doteq 6.6 + 5.0 + 1.0 \\ &\doteq 12.6 \text{ m}^2 \end{aligned}$$

About 12 m^2 of vinyl is needed to cover the foam in the hot tub.

I determined the area of the sides and the bottom for both the outside and the inside. The surface area of a cylinder is $SA = 2\pi r^2 + 2\pi rh$. The hot tub has a bottom but no top, so I used $SA = \pi r^2 + 2\pi rh$. The outside of the tub is 64 cm high and the inside is 61 cm deep, so I used 0.6 m for both.

I calculated the area of the top edge.

I added up all areas to estimate the total amount of vinyl required.

I used numbers that were mostly greater than the actual measurements. So I decided to round down.

Reflecting

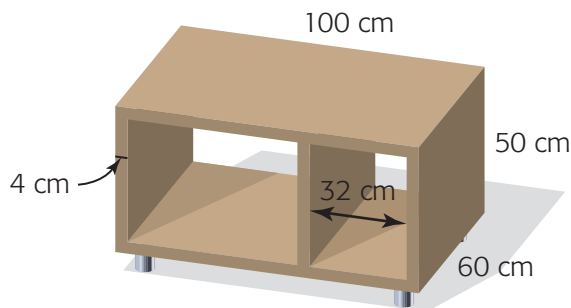
- Was Bay justified in estimating to solve the problem? Explain.
- How did Bay use logical reasoning to solve the problem?

WORK WITH the Math

EXAMPLE 2

Using logical reasoning to determine surface area

Nicole is building this bookshelf. She is enclosing one section with a door that is 32 cm wide and 2 cm thick. Determine the total area of wood she must finish with stain and varnish.



Nicole's Solution

1. Understand the Problem

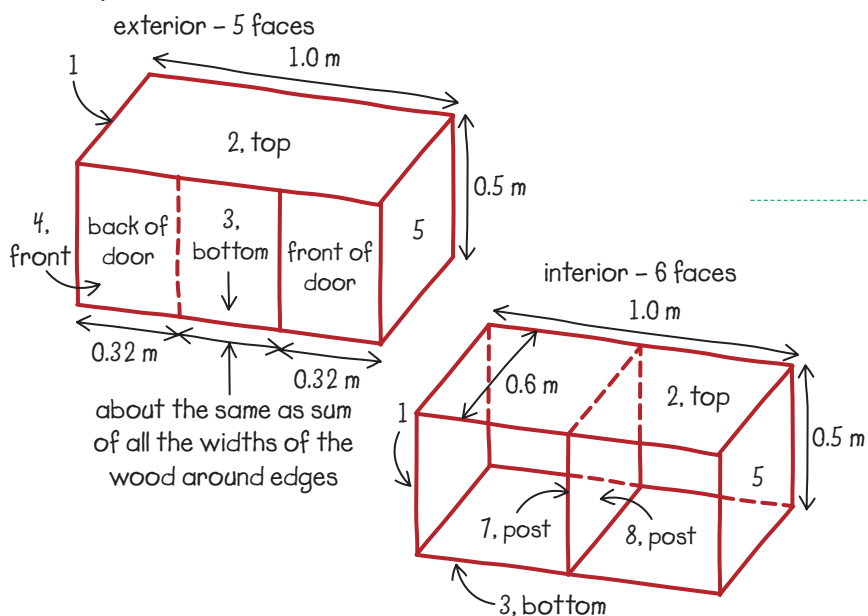
I'll stain every surface except the underside, which won't be seen. I can use the dimensions of the shelf to determine the area to stain.



2. Make a Plan

I should use measurements in metres rather than centimetres because varnish cans usually give coverage in square metres. This way I will avoid the conversion from square centimetres to square metres later. Also, I really only need an estimate, because I will buy the stain and varnish in cans, so I need to know only which size of can I need. I can solve a simpler problem to get my estimate.

3. Carry Out the Plan



The door is to be finished on both sides, so I visualized the bookshelf as having two doors, but being finished on only one side. The gap left over would be about the same area as the edges of the doors.

To make my estimate easier, I decided to use the same measurements for interior and exterior, even though I knew the interior length and height were less than the exterior length and height because of the 4 cm width of the wood.

$$\begin{aligned}
 \text{Surface area of exterior} &= 2(\text{face 1}) + 2(\text{face 2}) + (\text{face 4}) \\
 &= 2(0.6 \times 0.5) + 2(1.0 \times 0.6) \\
 &\quad + (1.0 \times 0.5) \\
 &= 2(0.3) + 2(0.6) + 0.5 \\
 &= 0.6 + 1.2 + 0.5 \\
 &= 2.3 \text{ m}^2
 \end{aligned}$$

Faces 1 and 5 are the same; faces 2 and 3 are the same.

$$\begin{aligned}
 \text{Surface area of interior} &= 4(\text{face 1}) + 2(\text{face 2}) \\
 &= 2(0.6) + 1.2 \\
 &= 1.2 + 1.2 \\
 &= 2.4 \text{ m}^2
 \end{aligned}$$

I had already calculated the area of two of face 1 and two of face 2.

$$\begin{aligned}
 \text{Total surface area} &= \text{exterior} + \text{interior} \\
 &= 2.3 + 2.4 \\
 &= 4.7 \text{ m}^2
 \end{aligned}$$

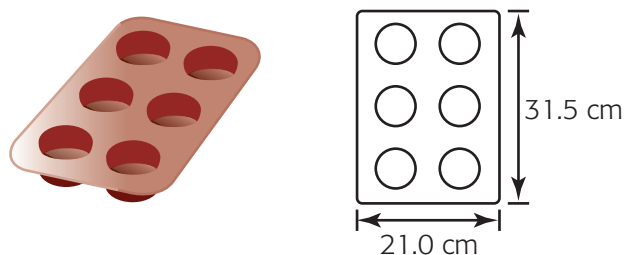
I need enough stain and varnish to coat about 5 m².

4. Look Back

My estimate is greater than the actual value, because the area of the sums of all the widths of the wood plus the front and back of the door is probably less than the full face 4 that I visualized. Also, I used greater measurements for the interior than the actual values. However, it's better to estimate a little high and have enough of the stain and varnish than to estimate low and not have enough.

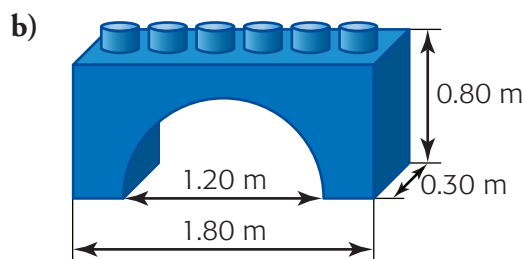
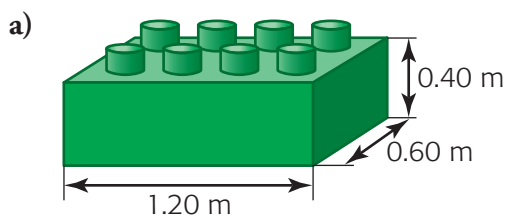
Checking

1. A silicone muffin pan is used to make cylindrical muffins with a 7.5 cm diameter. The pan is 4.0 cm deep. Calculate the surface area of silicone poured into a mould to make the muffin pans.

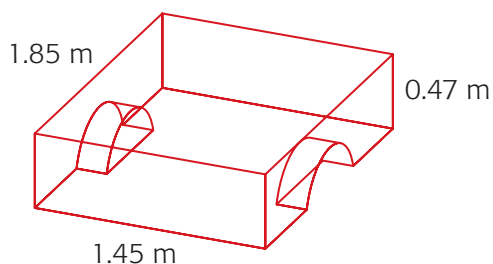


Practising

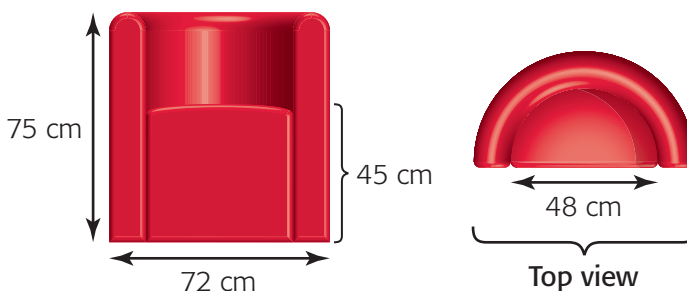
2. Johanna builds sets for her school theatre. One set uses giant linking cubes. Each cylindrical connector is 0.20 m in diameter and 0.15 m high. Calculate the total area Johanna must paint for each piece.



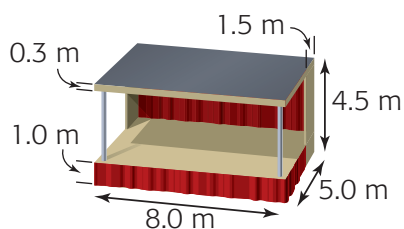
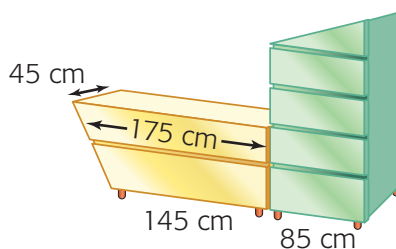
3. Alexis wants to spray a protective coating on the interior of her truck's cargo box. The box is rectangular with semicircular wheel wells, having a diameter of 0.60 m. It is 1.08 m between the wheel wells. Determine the total area that Alexis must spray.



4. A semicircular tub chair is upholstered in one fabric. Calculate the amount of fabric used to cover the chair.



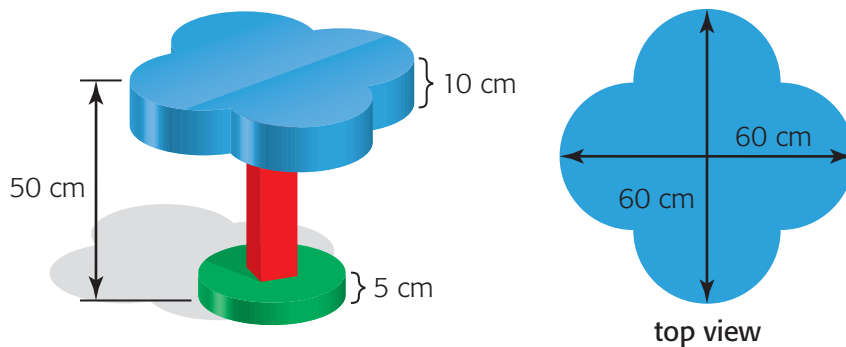
5. Benjamin designs modern furniture. He finishes each piece with colourful paint. Each drawer has a height of 24 cm. What surface area must he paint?



6. An outdoor stage is to be built as shown. The back and side walls are 10 cm thick. The cylindrical posts are 20 cm in diameter. Determine the surface area of the stage, not including the area touching the soil.

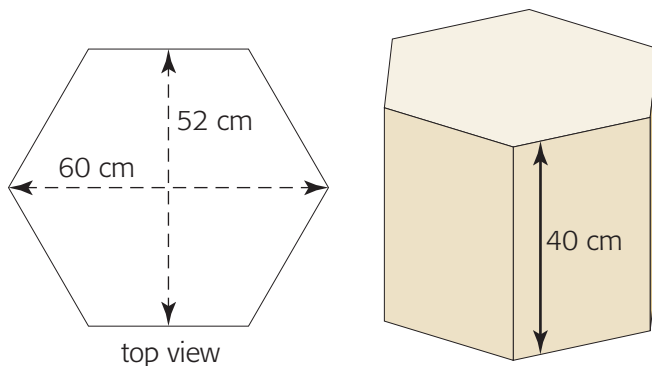
Closing

7. A child's table is built with three separate pieces as shown. The base has a diameter of 30 cm. The post is square: 8 cm each side. Determine the surface area of the whole table, including the base. Explain your reasoning.



Extending

8. A leather stool is in the shape of a regular hexagon. Calculate the amount of leather required to cover the entire stool.



9. When this alarm clock goes off, three objects pop out. To turn off the alarm, you need to replace the objects. The hole for each object is 2 cm deep.
- The green cylinder is 5 cm in diameter. The base of the blue rectangular prism is 5 cm by 3 cm. The legs of the base of the red right triangular prism are each 4 cm. Each object is 5 cm high. Determine the total surface area of each object.
 - Determine the total surface area of the alarm clock when the objects are in place.

