

**GOAL**

**Determine the area of overlap for the components of 3-D objects.**

**LEARN ABOUT the Math**

Amanda is building a lamp in her woodworking class, using three different pieces of wood as shown. She will glue the bases of the top two pieces to attach them to the piece underneath.

**? What is the total area of the surfaces that will be glued?**

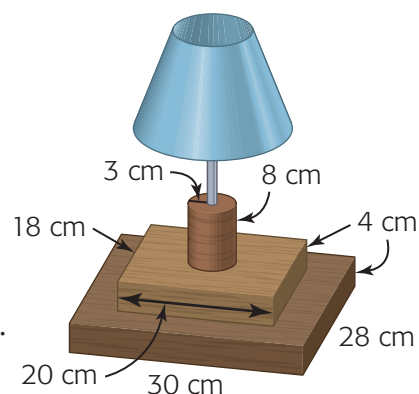
- Build a model of the lamp by stacking the prisms and the cylinder.
- Examine the model. Trace around the cylinder onto the smaller rectangular prism, then trace around the smaller prism onto the larger prism.
- Label on your diagrams the **areas of overlap** with their dimensions.
- What is the total area of overlap?

**Reflecting**

- Why might you need to know the area of overlap if you were making 100 of these lamps?
- When you are determining the area of overlap for two objects, do you have to think about the shapes of both objects?

**YOU WILL NEED**

- two different rectangular prisms
- a cylinder
- linking cubes
- isometric dot paper

**area of overlap**

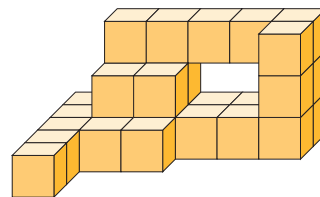
the area covered when two component parts are joined to form a composite object

## WORK WITH the Math

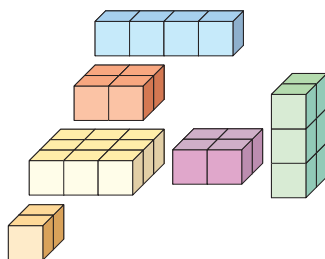
### EXAMPLE 1

### Building and decomposing a model

Decompose this structure into rectangular prisms and determine the total area of overlap. Each edge of each cube is 2 cm long.

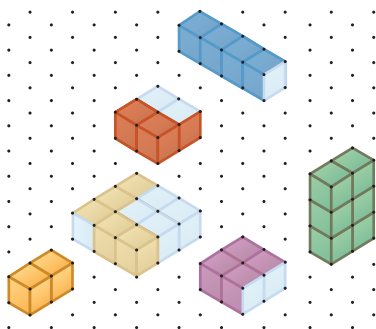


### Shelby's Solution



I decomposed the structure like this.

I used linking cubes of the same colour for each prism.



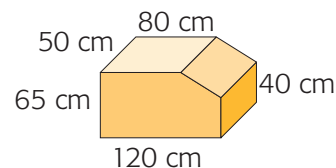
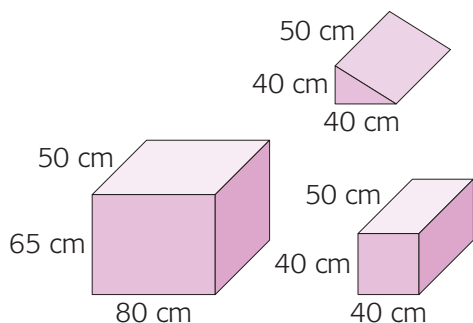
I sketched each prism on isometric paper and shaded one side of each connecting surface. I counted the shaded surfaces.

12 square faces on the prism surfaces are connected to other prisms, so the total area of overlap is  $12 \times 4 = 48 \text{ cm}^2$ .

I knew the face of each cube has an area of  $2 \times 2 = 4 \text{ cm}^2$ .

**EXAMPLE 2****Decomposing an object using different prisms**

Decompose this object and determine the area of overlap.

**Bay's Solution**

I visualized separating the object into two rectangular prisms and a triangular prism. I determined the dimensions of the prisms using given measurements and calculations.

The height of the original object is 65 cm, and the height of the small rectangular prism is 40 cm. So the height of the triangular prism is  $65 - 40 = 25$  cm.

The base of the original object is 120 cm long, and the large rectangular prism is 80 cm long. So the small rectangular prism and the triangular prism are  $120 - 80 = 40$  cm wide.

Area of overlap:

Right side of large rectangular prism

$$\begin{aligned} A &= l \times w \\ &= 65 \times 50 \\ &= 3250 \text{ cm}^2 \end{aligned}$$

Top of small rectangular prism

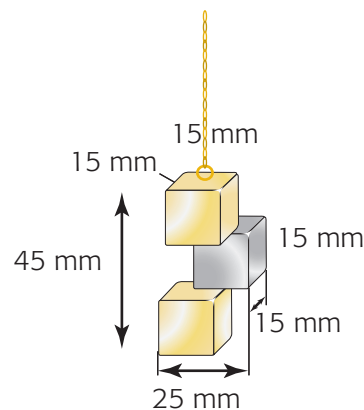
$$\begin{aligned} A &= l \times w \\ &= 40 \times 50 \\ &= 2000 \text{ cm}^2 \end{aligned}$$

I noticed that the small rectangular prism and the triangle overlap the complete side of the large rectangular prism. The triangular prism overlaps the top side of the small rectangular prism.

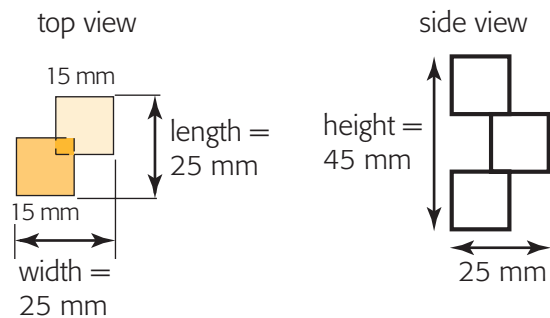
The total area of overlap is  $3250 + 2000 = 5250 \text{ cm}^2$ .

**EXAMPLE 3****Determining partial area of overlap**

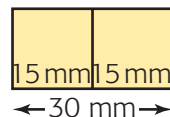
Brenda makes custom jewellery from metal cubes that are 15 mm on each side. This design has three cubes, as shown. What is the area of overlap?



## Yvonne's Solution



I knew that if I placed two metal cubes side by side, the total width would be 30 mm.



But the design is 25 mm wide, so I could calculate the width of the overlap. The length of the design was also 25 mm.

The cubes overlap by  
 $30 - 25 = 5$  mm in both directions.

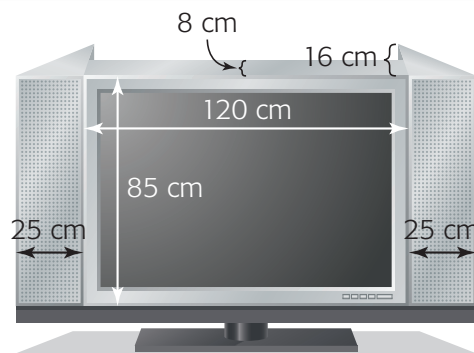
$$\begin{aligned}\text{Area of overlap per cube} &= (5)^2 \\ &= 25 \text{ mm}^2\end{aligned}$$

$$\begin{aligned}\text{Total area of overlap} &= 2 \times 25 \\ &= 50 \text{ mm}^2\end{aligned}$$

Each cube overlapped another cube by a square that is 5 mm by 5 mm. There were two identical areas of overlap.

### EXAMPLE 4 Determining area of overlap

Speakers in the shape of triangular prisms are attached on either side of a television screen. The entire unit is then connected to a base. Determine the total area of overlap.



## Derek's Solution

$$\begin{aligned}\text{Area of overlap between screen and speakers} &= 2 \times (l \times w) \\ &= 2 \times (85 \times 8) \\ &= 1360 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Area of overlap between screen and base} &= 120 \times 8 \\ &= 960 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Area of overlap between base and both speakers} &= 2 \times \left(\frac{1}{2}(b \times h)\right) \\ &= 2 \times \left(\frac{1}{2}(25 \times 16)\right) \\ &= 400 \text{ cm}^2\end{aligned}$$

First, I decided to determine the area of overlap between the television screen and the speakers. The screen is 85 cm high, and 8 cm deep. Its edges are rectangular. There is an area that size on either side of the screen. I also wanted to determine the area where the screen touches the base. The width of the screen is 120 cm.

Next, I needed to calculate the area of overlap between the two speakers and the base. They overlap in right triangular shapes. The base of each triangle is 25 cm, and the height is 16 cm.

Total area of overlap ..... I added all of the areas of overlap together.

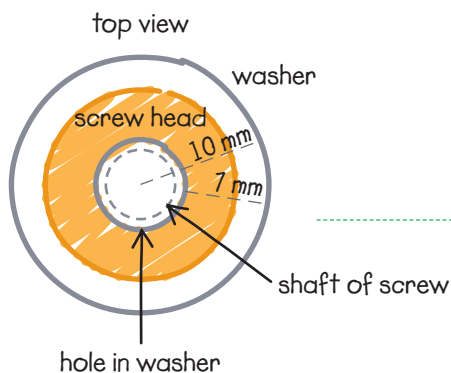
$$\begin{aligned}
 &= \text{area between screen and speakers} \\
 &\quad + \text{area between screen and base} \\
 &\quad + \text{area between base and speakers} \\
 &= 1360 + 960 + 400 \\
 &= 2720 \text{ cm}^2
 \end{aligned}$$

### EXAMPLE 5 Determining circular area of overlap

A washer is to be used with a screw. What is the area of overlap between the washer and the screw head?



### Amanda's Solution



I drew a diagram showing how the washer and screw would line up.

The area of overlap between the washer and the screw head is an outer circle the size of the screw head, with an inner circle the size of the hole in the washer removed.

I knew that the outer radius is half of the outer diameter of the screw head.

I calculated the radius of the hole in the washer.

I need to calculate the area of the orange part in my diagram.

Washer dimensions:

$$\text{Full radius} = 10 \text{ mm}$$

$$\begin{aligned}
 \text{Radius of hole} &= 10 - 7 \\
 &= 3 \text{ mm}
 \end{aligned}$$

$$\begin{array}{ll}
 \text{Area of screw head} & \text{Area of washer hole} \\
 = \pi r^2 & = \pi r^2 \\
 = \pi (7)^2 & = \pi (3)^2 \\
 = \pi 49 & = \pi 9
 \end{array}$$

I determined expressions for the area of the outer circle (the screw head) and the inner circle (the washer hole).

$$\begin{aligned}
 \text{Area of overlap} &= \pi 49 - \pi 9 \\
 &= \pi 40 \\
 &\doteq 126 \text{ mm}^2
 \end{aligned}$$

I subtracted to determine the area of overlap.

The area of overlap is about  $126 \text{ mm}^2$ .

## In Summary

### Key Ideas

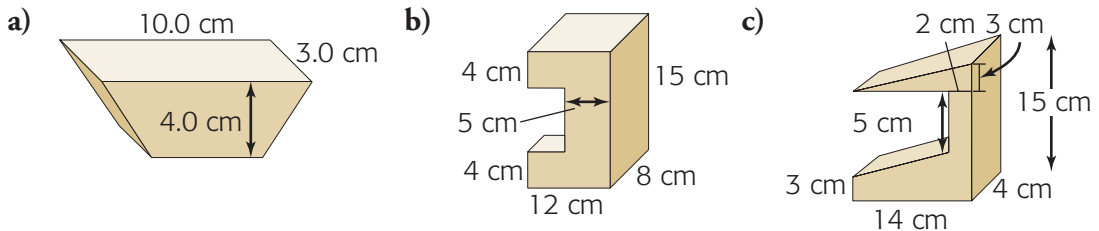
- When two objects are combined, there will be an area of overlap.
- The way in which a composite object is decomposed may affect its area of overlap.

### Need to Know

- The area of overlap can be all or part of the face of a component part.

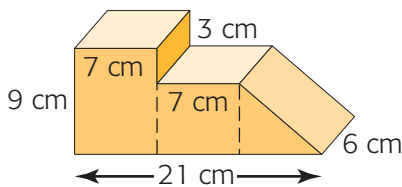
## Checking

1. Sketch component parts of each object. Indicate the areas of overlap.

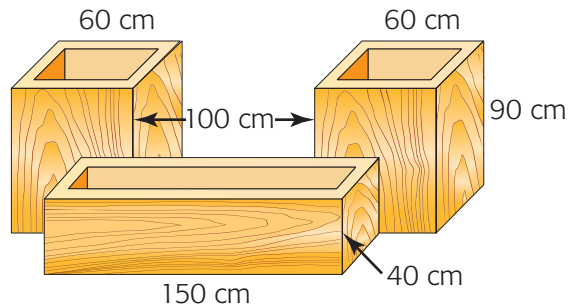


2. Calculate the total area of overlap for the components you identified in question 1.

## Practising

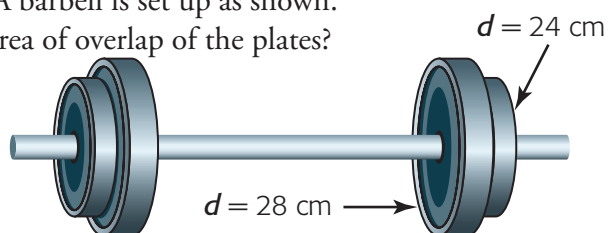


3. This object is decomposed into two rectangular prisms and a triangular prism, as shown. Calculate the area of overlap.
4. **Multiple choice.** A set of three planters is assembled as shown. What is the area of overlap of the exteriors of the planters?  
**A.** 2000 cm<sup>2</sup>   **B.** 2400 cm<sup>2</sup>   **C.** 2500 cm<sup>2</sup>   **D.** 4000 cm<sup>2</sup>

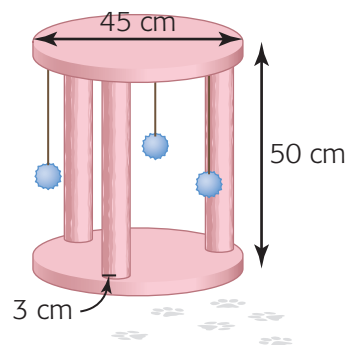
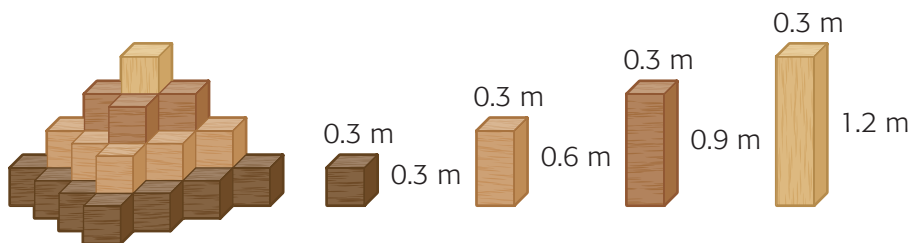


5. **Multiple choice.** A barbell is set up as shown. What is the total area of overlap of the plates?

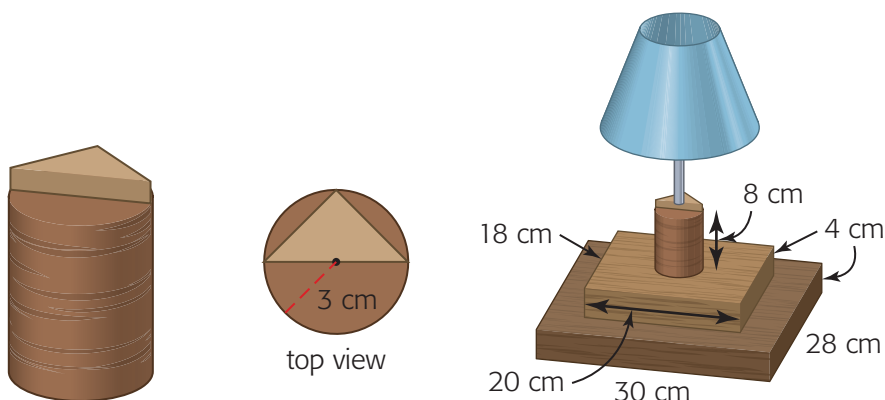
- A. 3620 cm<sup>2</sup>
- B. 452 cm<sup>2</sup>
- C. 1232 cm<sup>2</sup>
- D. 904 cm<sup>2</sup>



6. Calculate the areas of overlap in this cat toy. Each cylindrical post has a radius of 3.0 cm.
7. A store display is designed with rectangular prisms, as shown. The top of each prism is a square 0.3 m on each side. Calculate the total area of overlap of the prisms.



8. Amanda decides to add a fourth triangular piece of wood to the top of her lamp as shown. How would you determine the area of overlap between the new triangular prism and the cylinder?



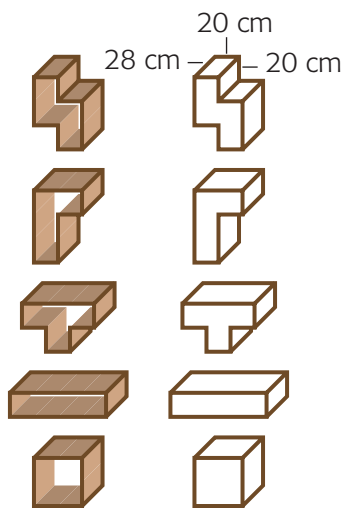
### Reading Strategy

#### Predicting

Use sketching and decomposing to help predict and confirm the area of overlap.

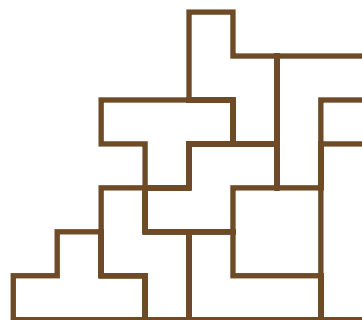
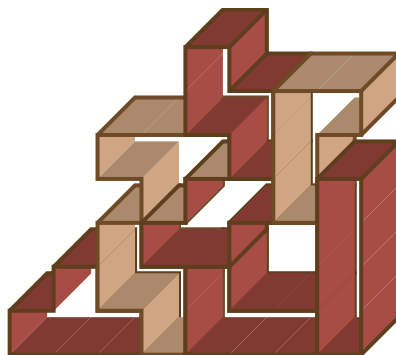
9. Build a composite structure with linking cubes. Exchange your object with a partner. Decompose your partner's object and determine the area of overlap.
10. A Kransekake is a Norwegian wedding cake. It is constructed using cookie rings that increase in diameter from top to bottom. Meaghan wants to bake a Kransekake using six cookie rings. The outer diameters for the cookies are 6.0 cm, 7.0 cm, 8.0 cm, 9.0 cm, 10.0 cm, and 11.0 cm. The inner diameter of each cookie ring is 2 cm less than its outer diameter. Calculate the total area of overlap of the cookies when assembled.





11. A wall unit is made using objects from a popular video game, as shown. Each object consists of four rectangular prisms that are 20 cm long, 20 cm wide, and 28 cm high.

a) This wall unit was built using nine of these objects. Calculate the area of overlap.



Front view

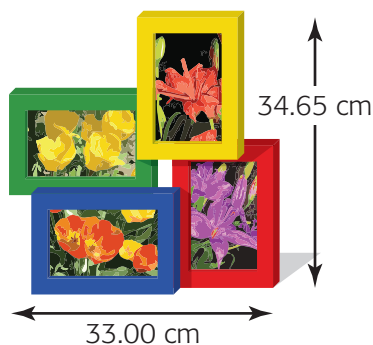
- b) Use linking cubes to create your own wall unit with 12 of these objects. Calculate the area of overlap.
- c) Exchange your design with a partner and calculate the area of overlap in your partner's design.

## Closing

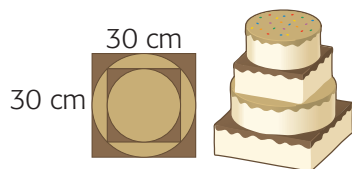
12. Is it possible for the overlap when two rectangular prisms touch to be the same as the overlap when two triangular prisms or two cylinders touch? Explain.

## Extending

13. This picture frame is made using four individual frames, as shown. This frame is 34.65 cm high and 33.00 cm long. Each individual frame is 17.80 cm by 12.60 cm, with a window of 14.00 cm by 8.80 cm. Also, each individual frame overlaps each other frame by  $\frac{1}{2}$  of the width of the frame. The bottom edges of the two lower frames are even.



- a) Determine the dimensions of the open space between the frames.
- b) Determine the area of overlap of the four individual frames.



14. A party cake consists of four layers, each the same height. There are two square layers, and two circular layers, as shown. Determine the area of overlap to the nearest square centimetre.