3.3 Scale Factors

YOU WILL NEED

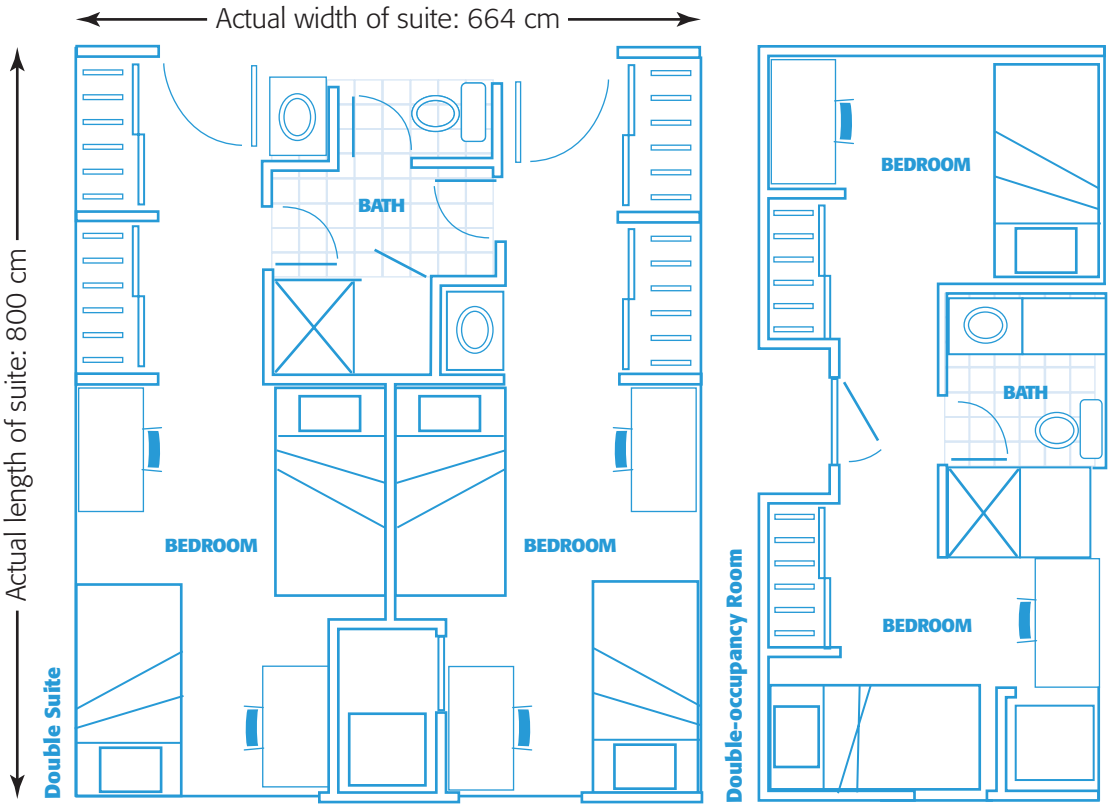
- a ruler
- a protractor
- a calculator

GOAL

Describe the relationship between similar shapes using a scale factor.

LEARN ABOUT the Math

Thomas went to the architect's talk on career day. He had a chance to look at the blueprints that builders use to construct buildings.



? If a measurement is missing on the blueprint, how can the actual measurement be determined?

A. Complete the table to compare the dimensions of the blueprint with the floor plan of the actual building. What do you notice?

	Blueprint	Floor Plan	Ratio
Length of double suite	10 cm	800 cm	
Width of double suite	8.3 cm		

B. Why is it important that the blueprint be similar to the actual floor plan?

C. What is the **scale factor** of the blueprint and the building?

D. Explain how the scale factor relates the length of the double suite on the blueprint to that of the building.

E. Use the scale factor to determine the width of the double-occupancy room in the building.

scale factor

the factor one dimension of a polygon is multiplied by to calculate the corresponding dimension of a similar polygon

Reflecting

F. If you know two shapes are similar, how can you calculate the scale factor?

G. If a measurement is missing on diagrams of similar shapes, how can you use the scale factor to calculate the missing measurement?

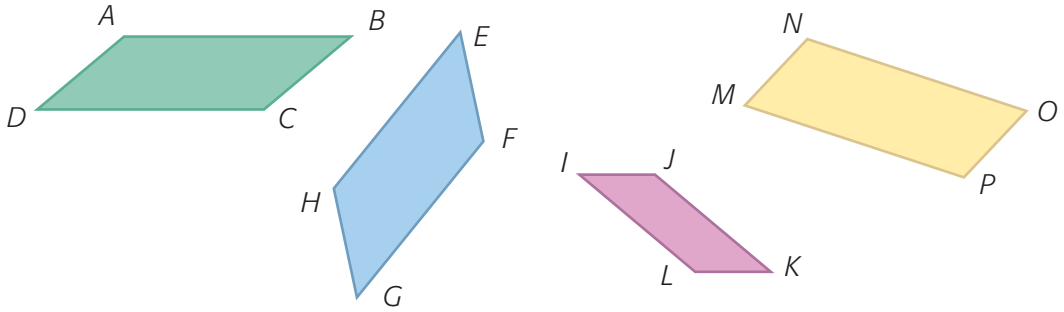
Communication *Tip*

The scale factor can be expressed as a ratio, a percent, or as a whole number, fraction, or decimal. For example, if lengths are doubled, the scale ratio can be described as 2:1, 200%, or 2.

WORK WITH the Math

EXAMPLE 1 Using scale factors to determine similarity

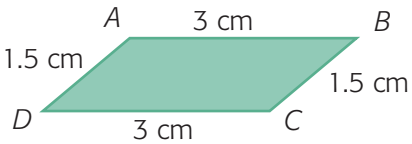
Determine which of these shapes are similar. For the shapes that are similar, determine the scale factor.

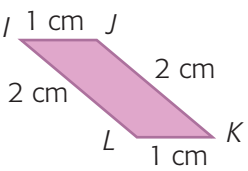


Rachel's Solution

$\angle DAB = 140°$ $\angle EFG = 130°$ $\angle IJK = 140°$ $\angle MNO = 115°$
$\angle ABC = 40°$ $\angle FGH = 50°$ $\angle JKL = 40°$ $\angle NOP = 65°$
$\angle DCB = 140°$ $\angle GHE = 130°$ $\angle KLI = 140°$ $\angle OPM = 115°$
$\angle CDA = 40°$ $\angle HEF = 50°$ $\angle LIJ = 40°$ $\angle PMN = 65°$

I measured all of the angles. The only two parallelograms with all of the corresponding angles the same are *ABCD* and *IJKL*. They could be similar.





I measured the side lengths for *ABCD* and *IJKL*.

$AB : JK = 3:2$ or $1.5:1$ $\quad AD : JI = 1.5:1$
$DC : IL = 3:2$ or $1.5:1$ $\quad BC : KL = 1.5:1$

I determined the ratios of the corresponding sides.

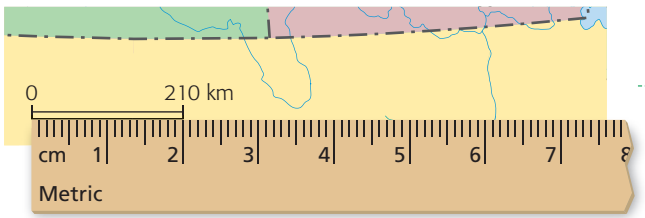
parallelogram $IJKL$ ~ parallelogram $ABCD$.
The scale factor for the enlargement of $IJKL$ to $ABCD$ is $3:2$ or 1.5 or 150%.
The scale factor for the reduction of $ABCD$ to $IJKL$ is $2:3$ or $\frac{2}{3}$ or about 67%.

I could also write the scale factor for the enlargement as $\frac{3}{2}$ or $1.5:1$.

EXAMPLE 2 Scale factors greater than 1

Thomas is planning a trip from Regina to Edmonton for a hockey tournament. He wants to know how long the trip will take.

Thomas's Solution



The highway from Regina to Winnipeg is not straight, but it is close, so I can use a ruler on the map to estimate the distance from Regina to Winnipeg.

The scale says 210 km is represented by 2 cm on the map. To figure out what to multiply by, the measurements have to be in the same units. I know that 210 km is 21 000 000 cm, so the scale factor is 21 000 000 : 2, or 10 500 000.

I used the relationship between units to calculate the scale factor.
I expect the scale factor to be greater than 1, since the actual distance is an enlargement of the map.

Regina to Winnipeg: 5.5 cm

I measured the distance from Regina to Winnipeg on the map.

$5.1 \times 10\,500\,000 = 53\,550\,000$ cm
$53\,550\,000 \div 100 = 535\,500$ m
$535\,500 \div 1000 = 535$ km

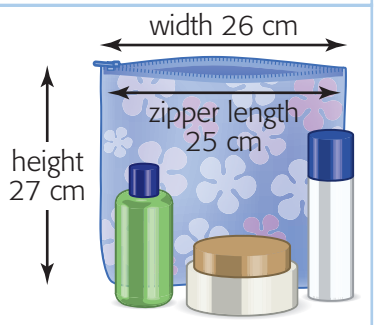
I multiplied by the scale factor to calculate the real distance.

535 km is close to 500 km and I will travel at about 100 km/h. I estimate the trip will take about 5 hours.

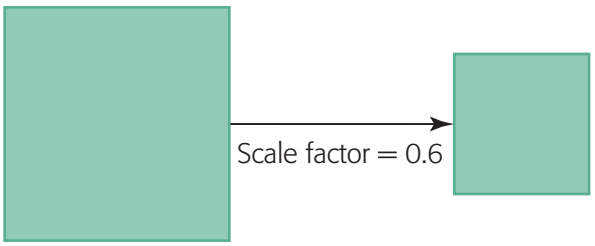
I rounded the distance and estimated my travelling speed to estimate how long the trip will take.

EXAMPLE 3 | Scale factor less than 1

Art students are making bags in their textile course. Their assignment is to make new bags that are 60% of the size of the pattern shown, but maintain the original shape. What will be the reduced measurements?



David's Solution: Using decimals



The new dimensions will be similar to the original dimensions. I expect the scale factor to be between 0 and 1, since the new bag is a reduction of the pattern. The scale factor is 0.6, since the new bag is 60% as wide.

$27 \times 0.6 = 16.2$
$26 \times 0.6 = 15.6$
$25 \times 0.6 = 15$

I multiplied each measurement by the scale factor 0.6 to calculate the new measurement.

The reduced dimensions will be 16.2 cm by 15.6 cm. The zipper will be 15 cm long.

Larissa's Solution: Using equivalent ratios

The scale factor is 6 : 10
For the 27 cm side:

$$\frac{6}{10} \overset{\times ?}{=} \frac{\square}{27}$$

$\times 2.7$

$10 \times 2.7 = 27$
$6 \times 2.7 = 16.2$

The length and width of the new bag will be 60% as long as the old size. As a ratio, that's 60 : 100 or 6 : 10.

The ratio of the new measurements to the old measurements will be equivalent to $\frac{6}{10}$.

For the 26 cm side:

$$\frac{6}{10} = \frac{\square}{26}$$

$10 \times 2.6 = 26$
$6 \times 2.6 = 15.6$

For the 25 cm zipper:

$$\frac{6}{10} = \frac{\square}{25}$$

$10 \times 2.5 = 25$
$6 \times 2.5 = 15$

The reduced bag will be 16.2 cm by 15.6 cm. The bag will need a zipper 15 cm long.

In Summary

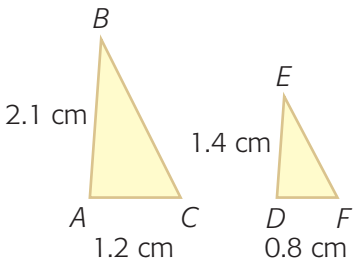
Key Ideas

- The ratio of a side length of a shape and the corresponding side length of a similar shape is the **scale factor**.
- You can multiply any length of a shape by the scale factor to calculate the corresponding length of a similar shape.

Need to Know

- When the scale factor is a number between 0 and 1, the new shape is a reduction of the original shape.
- When the scale factor is a number greater than 1, the new shape is an enlargement of the original shape.

Checking



1. $\triangle ABC \sim \triangle DEF$.

a) Write a statement using a scale factor that expresses one shape as an enlargement of the other.

b) Write a statement using a scale factor that expresses one shape as a reduction of the other.

2. Determine the missing measurement or scale factor.

	Scale Factor	Side Length	Corresponding Side Length
a)	4	6 cm	
b)	0.6		30 m
c)	25%	160 km	
d)		18 m	6 m

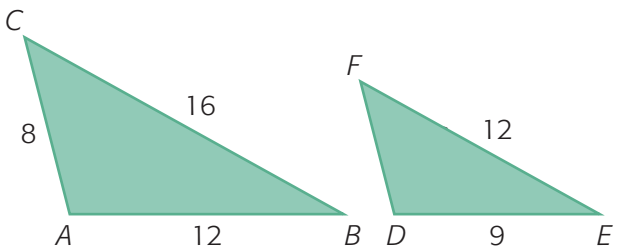
Practising

3. Complete the ratios for these similar triangles.

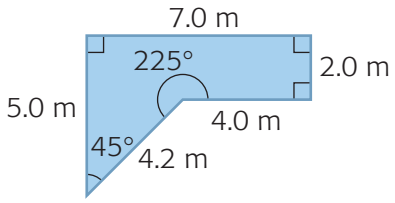
a) $\frac{AC}{DF} = \frac{?}{\text{DE}}$

b) $\frac{CB}{?} = \frac{\text{AB}}{\text{DE}}$

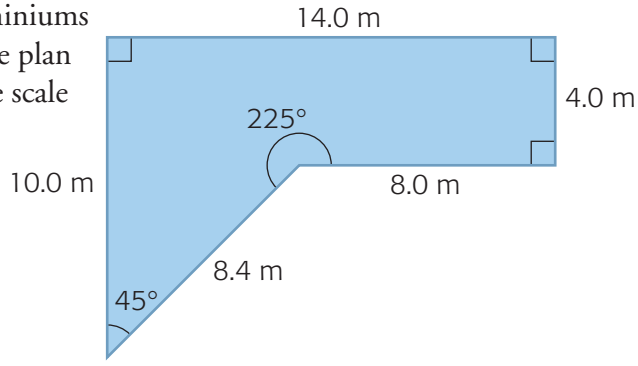
c) $\frac{8}{6} = \frac{16}{?}$



4. Kaycee saw two different ads about condominiums that are going to be built. Each ad shows the plan of the balcony in different sizes. What is the scale factor between the diagrams?

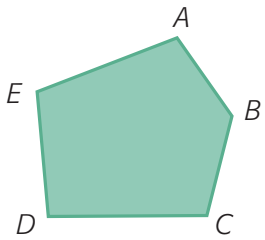


(A)



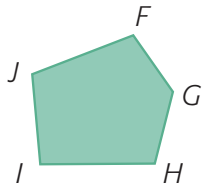
(B)

5. **a)** Explain why these polygons are similar.
b) Calculate the scale factor.



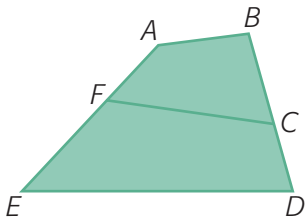
6. **Multiple choice.** To calculate the scale factor that reduced pentagon *ABCDE*

A. divide the length of *FG* by the length of *AB*
B. multiply the length of *DC* by the length of *IH*
C. multiply the length of *JI* by the length of *BC*
D. divide the length of *AE* by the length of *FJ*



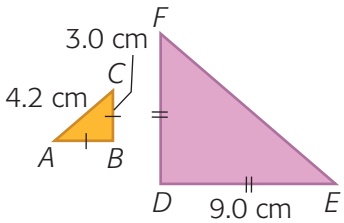
7. **Multiple choice.** A scale factor of a shape enlarged three and a half times can be represented by

A. 0.35 **B.** $\frac{3}{5}$ **C.** 350% **D.** 35



8. Determine whether quadrilaterals *ABDE* and *ABCF* are similar. Justify your answer using scale factor as part of your reasoning.

9. The ratios of the side lengths of $\triangle DEF$ to the corresponding side lengths of $\triangle ABC$ are 3 : 1. Determine the unknown lengths.

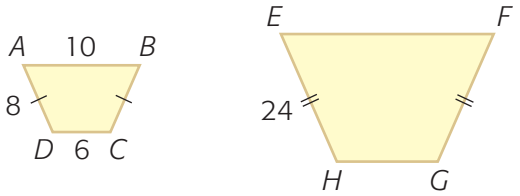


10. The scale on a map of a wilderness area shows that 2 cm represents 3 km. The camp store is 20 cm from the beach on the map.

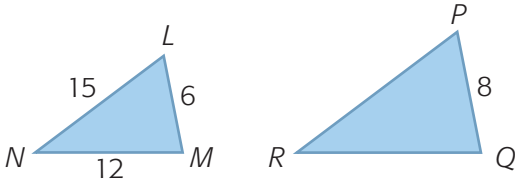
a) What is the scale factor between the map and the wilderness area?
b) How far apart are the camp store and the beach?

11. Use a scale factor to determine the missing side lengths for each pair of similar shapes.

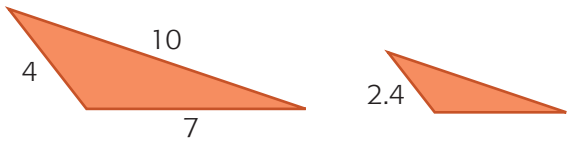
a)



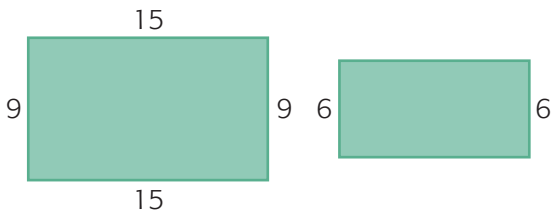
b)



c)



d)



12. An IMAX screen is eight stories high and measures 21.5 m by 15.6 m. A similar screen is 2.15 m long. What is the scale factor between the similar screen and the IMAX screen?

13. A software program offers these preset paper sizes for printing:
A4 (210 mm by 297 mm)
A5 (148 mm by 210 mm)
B5 (182 mm by 257 mm).

Use scale factors to determine if the paper sizes are similar.

14. **a)** Find an example of a diagram online that includes a scale factor. Calculate the actual measurements.
b) Use another scale factor to draw a new diagram.

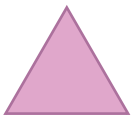


Figure 1

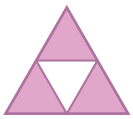


Figure 2

Figure 3

Figure 4

Closing

15. **a)** Draw a scale diagram of a place in your community or your home.
b) What scale factor did you use?
c) How did you decide which scale to use?

Extending

16. Fractal patterns demonstrate similarity. The triangles in each figure of the fractal pattern to the left are similar.
a) What is the scale factor between the triangle in Figure 1 and each triangle in Figure 2?
b) What is the scale factor between the triangles in Figure 2 and the triangles in Figure 3?
c) What visual pattern do you notice as the figure number increases?

17. Design a pictograph or symbol for scale factor that you could use in maps. Explain how it represents the idea of scale.