3.2 Determining Similarity

GOAL

Develop strategies to determine if two shapes are similar.

YOU WILL NEED

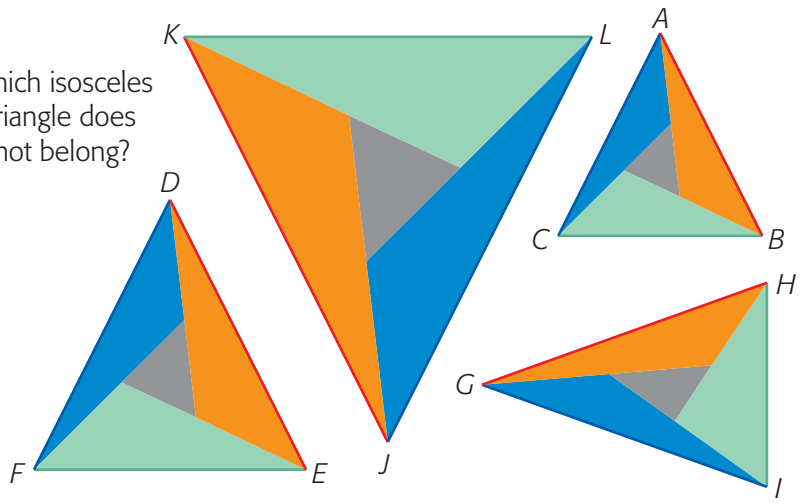
- a ruler
- a protractor

LEARN ABOUT the Math

Jia-Wen and Sam are sorting isosceles triangles to solve a puzzle in a magazine.

Sam thinks that triangle *GHI* is the answer.

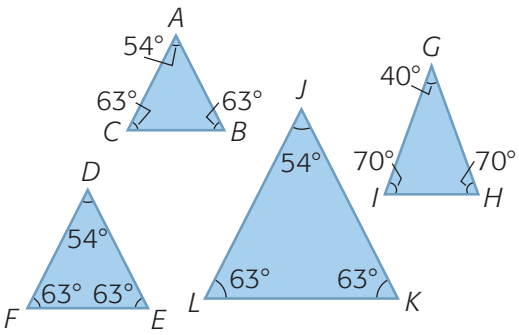
Which isosceles triangle does not belong?



How can you verify that Sam is correct?

EXAMPLE 1 Sorting shapes by properties

Sam's Solution: Comparing angles

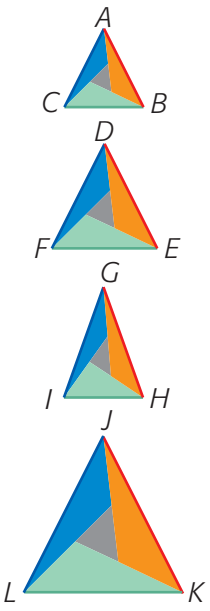


I noticed that the **corresponding angles** in the triangles with the same shape seem to be equal. I decided to measure the angles to see how they compared. I sketched the triangles to record the measurements.

corresponding angles

angles that match when two shapes are arranged to look the same; e.g., in Sam's triangles, $\angle ACB$ and $\angle DFE$ are corresponding angles.

$\triangle GHI$ does not belong. Its angles are 40°, 70°, and 70° and do not equal the corresponding angles in the other triangles.
All the other triangles have angles that are 54°, 63°, and 63°.

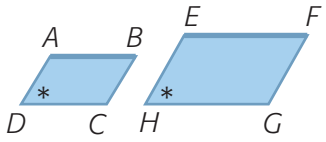


corresponding sides

sides that match if two shapes are arranged to look the same; e.g., in the parallelograms below, *AB* and *EF* are corresponding sides.

similar polygons

two or more polygons that are identical or where each polygon looks like an enlargement or reduction of the other. In similar polygons, the ratios of all corresponding linear measures are the same, and all corresponding angles are equal; e.g., in the similar polygons below, $\angle D$ and $\angle H$ are corresponding angles and *AD* and *EH* are corresponding sides.



Jia-Wen's Solution: Comparing sides

Triangle	Base (cm)	Side 1 (cm)	Side 2 (cm)
ABC	2.7	3	3
DEF	3.6	4	4
GHI	2.7	4	4
JKL	5.4	6	6

I measured the sides of each triangle and wrote the measurements in a table so that I could look for patterns.

For $\triangle JKL$ and $\triangle ABC$

Base: $LK : CB$

$5.4 : 2.7$

$2 : 1$

Side 1: $JK : AB$

$6 : 3$

$2 : 1$

Side 2: $JL : AC$

$6 : 3$

$2 : 1$

I noticed that the shape for $\triangle ABC$ and the shape for $\triangle JKL$ look the same. The measurements for $\triangle JKL$ are double the measurements for $\triangle ABC$.

This means the ratio of the bases is the same as the ratio of the sides.

The ratios of the lengths of **corresponding sides** are the same.

For $\triangle DEF$ and $\triangle ABC$

Base: $FE : CB$

$3.6 : 2.7$

$4 : 3$

Side 1: $DE : AB$

$4 : 3$

Side 2: $DF : AC$

$4 : 3$

I decided to compare the sides in other pairs of triangles that look like they have the same shape.

Since $\triangle ABC$, $\triangle DEF$, and $\triangle JKL$ all have the same ratios of corresponding sides, they are all **similar polygons**.

The ratios of corresponding sides are the same.

$\triangle ABC \sim \triangle DEF \sim \triangle JKL$

For $\triangle JKL$ and $\triangle GHI$

Bases: $KL : HI$
$5.4 : 2.7$
$2 : 1$

Side 1: $JK : GH$
$6 : 4$
$3 : 2$

Then, I compared $\triangle GHI$ to each of the other triangles to see if the ratios of corresponding sides were different.

The ratios of corresponding sides are different. $\triangle JKL$ and $\triangle GHI$ are not similar.

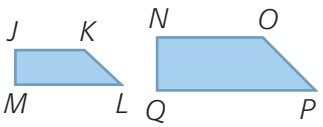
$\triangle DEF$ and $\triangle GHI$ are not similar. They have the same side lengths but different base lengths, so I know the ratios of corresponding sides are different.

$\triangle ABC$ and $\triangle GHI$ are not similar. They have the same base lengths but different side lengths, so I know that the ratios of corresponding sides are different.

$\triangle GHI$ does not belong because when you compare it to the other triangles, the corresponding sides form different ratios.

Communication Tip

The symbol ~ is used to show that two shapes are similar. For these two similar trapezoids, you can write JKLM ~ NOPQ. Notice that the first, second, third, and fourth letters in each trapezoid's name represent corresponding angles.



Reflecting

A. If two triangles have two pairs of corresponding sides in the same ratio, can you conclude that they are similar?

B. If all of the corresponding angles of a triangle are equal, the triangles are similar. How many pairs of corresponding angles do you need to show are equal before you can conclude that two triangles are similar?

C. Should you look for proportional sides or equal angles to test whether four-sided figures are similar? Explain.

Reading Strategy

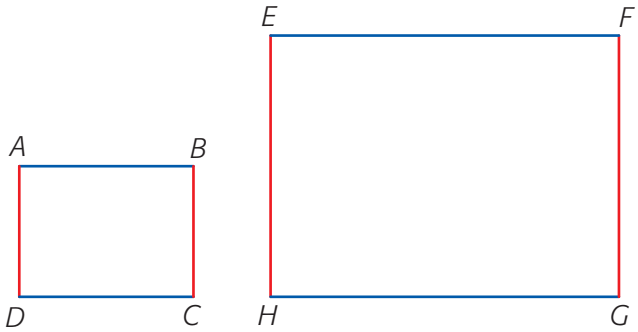
Activating Knowledge

What do you already know about polygons that will help you to answer these questions?

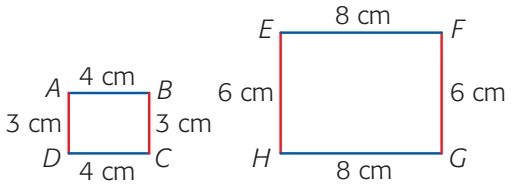
WORK WITH the Math

EXAMPLE 2 | Comparing ratios within shapes

Are these rectangles similar? If so, by how much is *ABCD* enlarged to get *EFGH*?



David's Solution



All rectangles have equal angles, so measuring the angles wouldn't help me decide. I measured and recorded the lengths and widths.

Ratios of lengths of corresponding sides:
$AB : EF \quad 4:8 = 1:2$
$AD : EH \quad 3:6 = 1:2$
The ratios are the same.
$ABCD \sim EFGH$
EFGH is twice as large as *ABCD*.

I didn't have to check *BC* : *FG* or *CD* : *GH* because in rectangles opposite sides are equal, so I knew the ratios would be the same.

Length : Width
$AB : AD \quad 4:3$

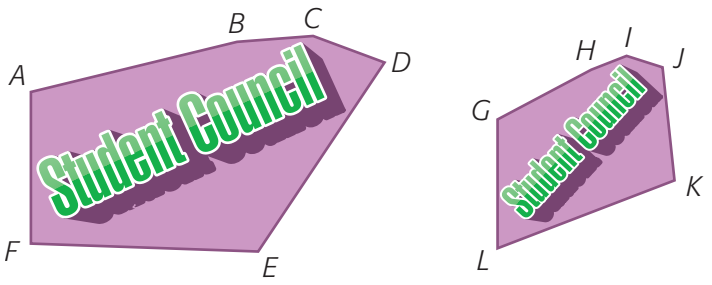
$EF : EH \quad 8:6 = 4:3$

While I was working on the solution, I noticed that the ratio of the length to width in each rectangle was 4 : 3.

I think I can compare the ratios of all corresponding sides between rectangles or I can compare the ratio of lengths and widths within rectangles to determine similarity.

EXAMPLE 3 | Checking for similarity

The student council sent in design *ABCDEF* and asked for a reduction of it to be put onto T-shirts. When the T-shirts arrived, the design looked like *GHIJKL*. How can they convince the company that an error has been made?



Larissa's Solution

$AB : GH$ $2 : 1$
$BC : HI$ $2 : 1$
$CD : IJ$ $2 : 1$
$DE : JK$ $2 : 1$
$EF : KL$ $3 : 2.5$, or $1.2 : 1$
$FA : LG$ $2 : 1$

I decided to write a letter to the company explaining why the designs are not the same shape.
I chose to measure, and then compare, the pairs of corresponding sides.

The two hexagons are not similar. Their corresponding angles must also not be equal.

$\angle AFE = 92°$ $\angle GLK = 69°$

To confirm my conjecture, I measured $\angle AFE$ and $\angle GLK$.

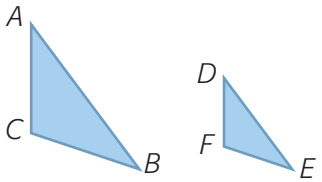
One pair of corresponding angles are not the same. My conjecture is correct.

When something should be true for all cases, I only have to show one case where it fails. Just showing that the angles were different would have been enough.

In Summary

Key Ideas

- Polygons are similar if the ratios of all pairs of corresponding sides are the same and all pairs of corresponding angles are equal. $\angle A = \angle D$, $\angle B = \angle E$, and $\angle C = \angle F$.
- The ratios of sides within one shape are the same as the ratio of the corresponding sides within a similar shape. $AB : BC = DE : EF$



(continued)

Need to Know

- To show that two polygons are similar, you must compare the ratios of the side lengths and check that corresponding angles are equal.

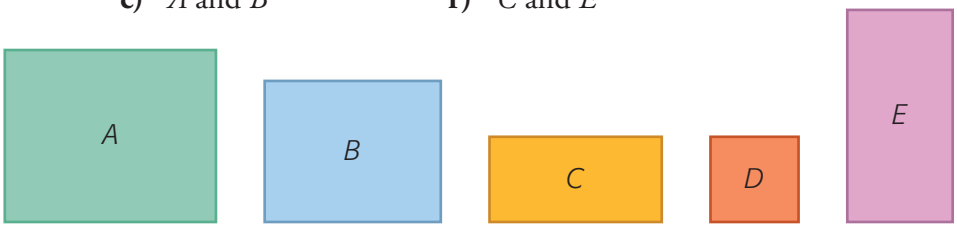
same angles, but not similar

corresponding sides in same ratio, but not similar

Checking

1. Is each pair of shapes similar? Explain how you know.

a) A and E
b) B and D
c) A and B
d) D and E
e) A and C
f) C and E



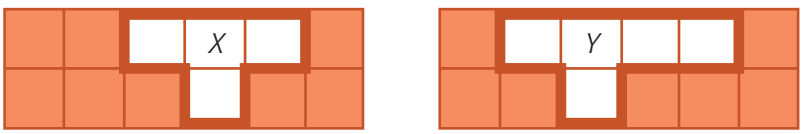
2. Three rectangles have these dimensions: 6 cm by 2 cm, 12 cm by 4 cm, and 18 cm by 6 cm. Are they similar? Explain your thinking.

Practising

3. Which shapes are similar?

4. Is each pair of shapes similar? Justify your answer.

a)



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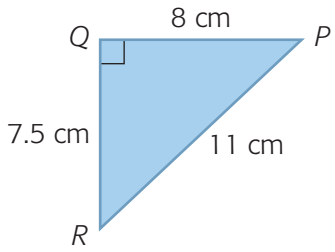

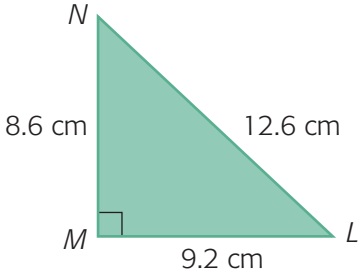

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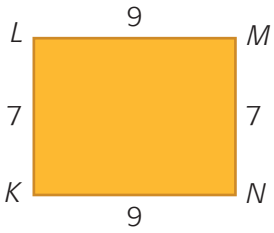
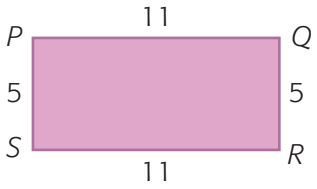
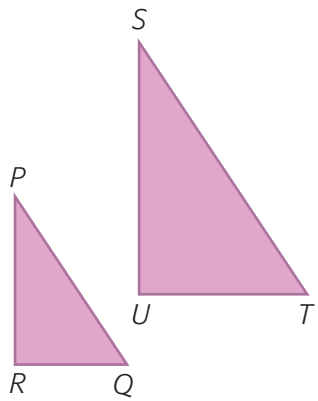




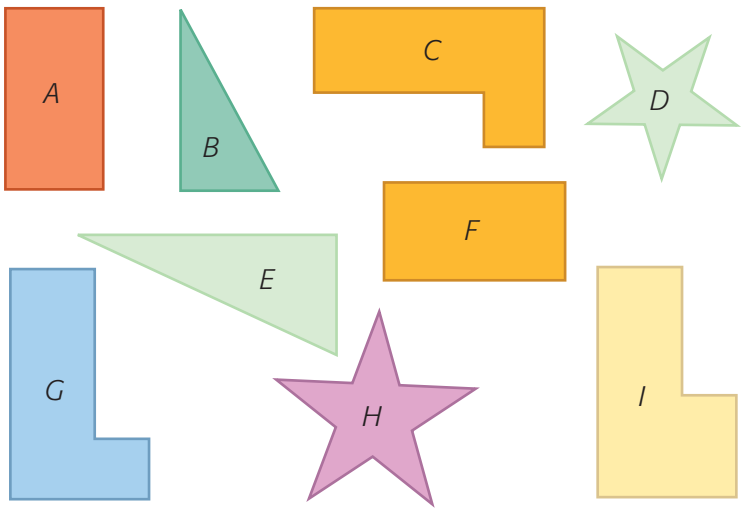
5. Select the similar shape for each.

	Shape	A	B	C	D
a)					
b)					
c)					

6. The triangles at right are similar. How can you prove it?

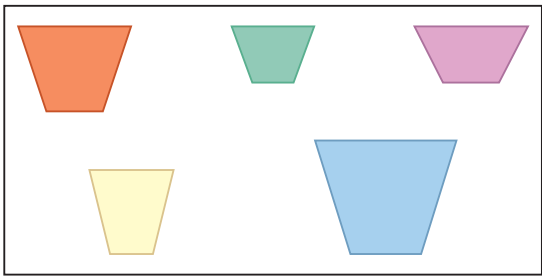


7. Which shapes are similar? Justify your decisions.

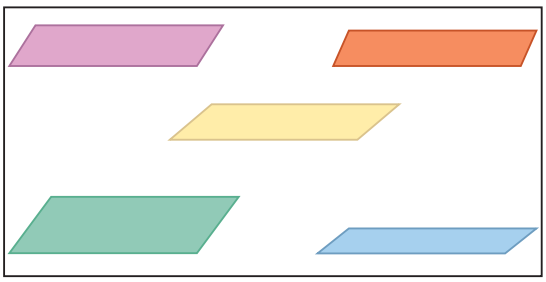


8. Multiple choice. Which set includes only similar polygons?

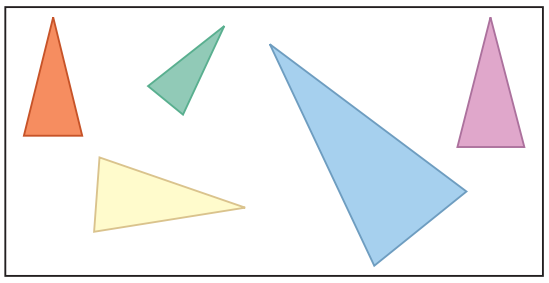
A.



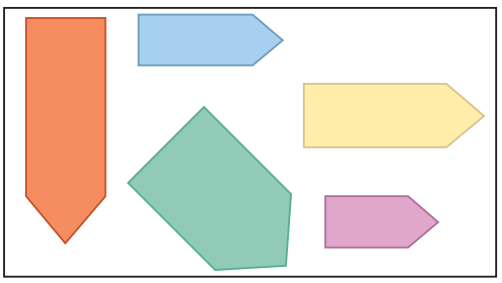
C.



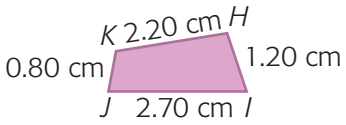
B.

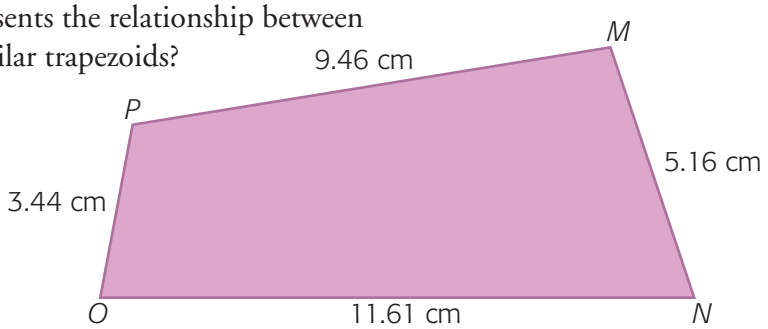


D.



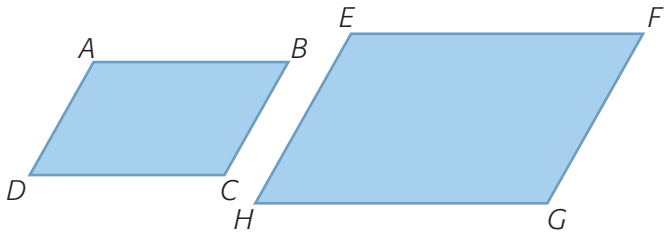
9. Multiple choice. Which ratio represents the relationship between the corresponding sides of these similar trapezoids?





A. 1 : 2.1 **B.** 1 : 4.3 **C.** 2 : 3 **D.** 1 : 3.4

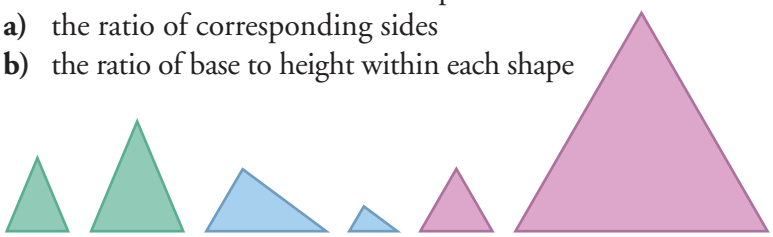
10. Measure the dimensions of the parallelograms and calculate the ratio of the lengths of adjacent sides *AB* and *AD* and *EF* and *EH*. What conclusion can you make?



11. Measure the sides of the similar shapes and determine

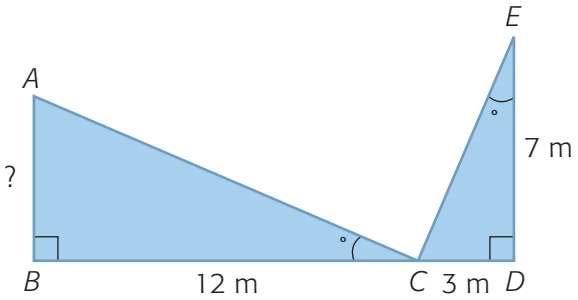
a) the ratio of corresponding sides

b) the ratio of base to height within each shape

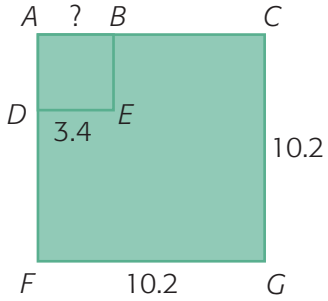


12. These sets of polygons are similar. What is the length of side *AB*?

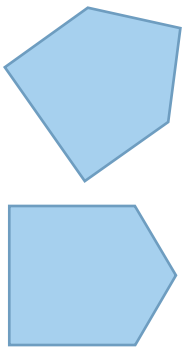
a)



b)



13. Measure the dimensions of the polygons at right and determine if they are similar. Explain your decision.



14. True or False? Explain your decisions.

a) All congruent shapes are similar.
b) All similar shapes are congruent.
c) All rectangles are similar.
d) All squares are similar.
e) All equilateral triangles are similar.
f) All isosceles triangles are similar.

15. With which statements do you agree? Why?
To decide if two polygons are similar, you need to measure

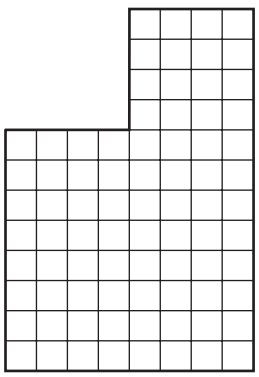
A. just two sides of each polygon
B. all the sides of each polygon
C. all the angles of each polygon
D. all the sides and all the angles

Closing

16. $\triangle ABC$ has angles of 45°, 45°, and 90°. $\triangle DEF$ has angles of 60°, 60°, and 60°. Are the triangles similar? Explain.

Extending

17. **a)** Draw the shape at right on grid paper.
b) Sketch four shapes inside that are similar to the original shape.



18. Suppose that $\triangle PQR \sim \triangle LMN$ and $\angle P = 90°$.

a) What angle in $\triangle LMN$ equals 90°? How do you know?
b) $MN = 13$ cm, $LN = 12$ cm, $LM = 5$ cm, and $PQ = 15$ cm. How long are PR and QR?

19. How would you convince someone that $\triangle ABC$ is similar to $\triangle DBE$?