1.5 Order of Operations with Rational Numbers

GOAL

Extend the order of operations rules to rational numbers.

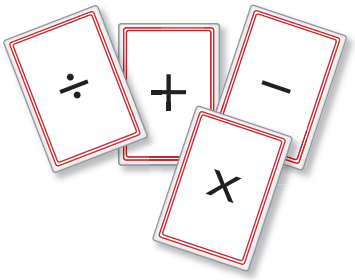
YOU WILL NEED

- a calculator with fraction capability

LEARN ABOUT the Math

Sam is solving a puzzle where he has to put in operations signs, $\times$, $-$, $\div$, and $+$, to make this statement true.

$$\frac{1}{2} \blacksquare (-2) \blacksquare \left(-\frac{3}{2}\right) \blacksquare \frac{1}{2} \blacksquare 5 = -16$$



? Which signs should Sam use?

A. How do you know that the sign before the 5 cannot be a $+$ sign?

B. Why is it more likely that you multiply by the 5 than divide by it?

C. How does knowing that the value is -16 help you know that the sign between -2 and $-\frac{3}{2}$ cannot be a $\times$ or $\div$ sign?

D. Where does each sign go?

Communication *Tip*

The convention for order of operations is BDMAS:
B: First calculate anything inside brackets.
DM: Divide and multiply from left to right.
AS: Add and subtract from left to right.

Reflecting

E. Why would you not need to know the order of operations rules if the $+$ sign were last?

F. Why do you need to know the order of operations rules if the $+$ sign is not last?

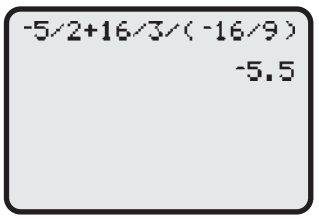
WORK WITH the Math

EXAMPLE 1 Using the order of operations to evaluate a rational-number expression

Evaluate $-2\frac{1}{2} + x \div y$ on a calculator when $x = 5\frac{1}{3}$ and $y = -1\frac{7}{9}$. Does your calculator follow the rules for order of operations?

Larissa's Solution

$-2\frac{1}{2} + x \div y \rightarrow \frac{-5}{2} + \frac{16}{3} \div \left(\frac{-16}{9}\right)$ I substituted the given values for the variables.



The answer on my calculator was -5.5, which is $-5\frac{1}{2}$.

$$-2\frac{1}{2} + 5\frac{1}{3} \div \left(-1\frac{7}{9}\right)$$
$$= \frac{-5}{2} + \frac{16}{3} \div \left(\frac{-16}{9}\right)$$
$$= \frac{-5}{2} + \left(\frac{-9}{3}\right)$$
$$= -2\frac{1}{2} + (-3)$$
$$= -5\frac{1}{2}$$

I calculated by hand to see if the calculator used the order of operations. I followed the order of operations the same way I would if the numbers were all integers or all fractions.

The calculator did follow the rules for order of operations.

EXAMPLE 2 Solving a problem using the order of operations

Thomas had been texting daily temperatures to Sam.

Temperatures	−0.2	+5.8	−5.1	+9.1

Thomas then told Sam how to calculate the mean.
He texted, "Add the numbers and divide by 4." Sam wrote:

$$-0.2 + 5.8 + (-5.1) + 9.1 \div 4 = -0.2 + 5.8 + (-5.1) + 2.275$$
$$= 2.775$$

2.775 is the average temperature.

What is wrong with Sam's calculation? What is the actual average?

Thomas's Solution

Sam's error was that he divided only the 9.1 by 4.

Sam should have used brackets around $-0.2 + 5.8 + (-5.1) + 9.1$ before dividing by 4.

Correction:

$$[-0.2 + 5.8 + (-5.1) + 9.1] \div 4$$

First I did the addition since the values are in brackets.

$$= [-5.3 + 14.9] \div 4$$
$$= 9.6 \div 4$$
$$= 2.4$$

I added the positives and the negatives separately.

The actual average is 2.4.

In Summary

Key Idea

- The rules for order of operations with rationals are the same as with integers:
  - Do what is in brackets first.
  - Multiply and divide from left to right.
  - Add and subtract from left to right.

Checking

1. Which operation would you perform first in each situation?

a) $2.4 - [3.5 + (-2.9)] \times (-5)$

b) $(-5.2 - 1.4) \div (-3) \times (2.1)$

2. Share values went up and down over the course of a week as shown at right. What was the mean daily change?

Day	Share Value
1	+0.005
2	−0.135
3	−0.115
4	−0.12
5	+0.05

Practising

3. Which operation would you perform last in each situation?

a) $3.7[3.5 + (-2.9)] \div [2.5 + (-1.8)]$

b) $8 \div (-1.3) + (-6.8 - 3.4)$

c) $\frac{2}{3} \times \frac{5}{4} - \frac{3}{4} \times \frac{9}{5}$

4. Temperatures went up and down over the course of five days as follows:
$-4.2\ °C, -1.4\ °C, +1.9\ °C, +3.7\ °C, -1.8\ °C$
What was the mean daily change?

5. Multiple choice. What is the value of
$[2.4 - 5.7] \times [-5.1 - (-1.8)] + 0.2$?

A. 11.09 **B.** 17.23 **C.** 17.83 **D.** 16.23

6. Multiple choice. What is the value of
$-0.7 - 0.3 \div (-0.15) \times 0.2 + 2$?

A. 0.4 **B.** 1.7 **C.** −0.46 **D.** −1.6090909...

7. Mika calculated $\left(-\frac{3}{4}\right) + \left(-\frac{2}{3}\right) \div \frac{1}{3}$ as $-4\frac{1}{4}$. Is Mika correct? Explain.

8. Evaluate.

a) $-3\frac{2}{3}x \div y$ when $x = 2\frac{1}{2}$ and $y = -3\frac{3}{4}$

b) $2x - \frac{5}{3}y$ when $x = -1\frac{3}{4}$ and $y = -\frac{7}{10}$

9. The temperature on Friday went up 3.1 °C from Thursday. After a loss of −1.2 °C on Saturday, the temperature doubled on Sunday. How did the temperature on Sunday compare to the temperature on Thursday?

10. For which calculations would you not need to know the order of operations rules?

a) $\left(-\frac{2}{3}\right) - \frac{5}{8} - \left(-\frac{4}{5}\right)$

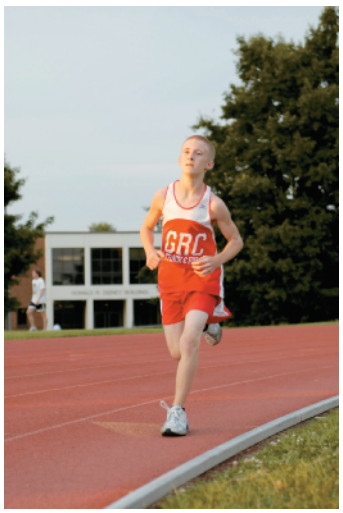
b) $\left(-\frac{2}{3}\right) \times \frac{5}{8} - \left(-\frac{4}{5}\right)$

c) $\left(-\frac{2}{3}\right) - \frac{5}{8} - \left(-\frac{4}{5}\right) \times 2$

d) $\left(-\frac{2}{3}\right) \div \frac{5}{8} \times \left(-\frac{4}{5}\right)$

11. a) Create an expression, involving both positive and negative rational numbers that are not integers, for which you would need to know the order of operations to calculate it correctly. Explain why you would need to know the order of operation rules for your expression.

b) Check to see if your calculator follows those rules for order of operations when you enter rational numbers. How do you know?



12. During the last heat for the 800 m run at the Jeux de la Francophonie in Edmonton, Xavier gained $\frac{1}{4}$ s over his nearest opponent during the first 100 m. He then lost $\frac{2}{3}$ s during the second 100 m, lost a further $\frac{1}{8}$ s during the third 100 m, but gained $\frac{1}{2}$ s during the fourth 100 m. How much time will Xavier have to gain on his opponent during the last 400 m to win the heat?

Closing

13. Why is it essential that the rules for order of operations for rational numbers be the same as the order of operations for integers?

Extending

14. Create an expression involving rational numbers where you perform an addition before you perform any multiplications or divisions. Calculate the value of that expression.

15. Where would you add brackets in each expression to make it true?

a) $3\frac{1}{2} + 4 \div 0.75 + (-8.1) = 1.9$

b) $-1.2 + -3 \div 1.5 + 1.5 - \frac{1}{5} \times 13 = -4$

16. You subtract a rational number from $-\frac{2}{3}$, double the answer, and then divide by $-\frac{1}{4}$. The result is 8. What was the rational number?