1.4 Multiplying and Dividing Rational Numbers

YOU WILL NEED

- a calculator

GOAL

Solve problems that involve multiplying and dividing rational numbers.

LEARN ABOUT the Math

Thomas was talking to his cousin in the United States. Thomas said the temperature was $-5.5°$. His cousin said that the temperature where he lived was $-1.5°$. However, Thomas was using the Celsius scale and his cousin was using the Fahrenheit scale.

Thomas found this conversion formula:

$$F = \frac{9}{5}C + 32$$

? Which temperature was lower?

A. What would Thomas's temperature be in Fahrenheit, if it had actually been $+5.5$ °C instead of -5.5 °C?

B. Why does it make sense that $\frac{9}{5} \times (-5.5)$ is the opposite of $\frac{9}{5} \times 5.5$?

C. Thomas's actual temperature was -5.5 °C. What was his temperature in degrees Fahrenheit?

D. What was his cousin's temperature in degrees Celsius?

E. Which temperature was lower?

Reflecting

F. Why were your answers to parts A and C not opposites?

G. How does knowing how to multiply and divide integers help you calculate the values in parts C and D?

H. How does knowing how to multiply and divide fractions help you calculate the values in parts C and D?

WORK WITH the Math

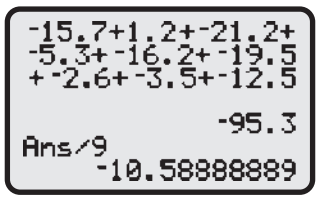
EXAMPLE 1 Reasoning about the mean of a set of rational numbers

These temperatures were recorded at noon on January 1 from 2000 to 2008 at Edmonton International Airport. Determine the mean noon temperature on January 1 for the years given.

Year	Temperature (°C)
2000	−15.7
2001	1.2
2002	−21.2
2003	−5.3
2004	−16.2
2005	−19.5
2006	−2.6
2007	−3.5
2008	−12.5

Source: Environment Canada

Rachel's Solution



I used my calculator and added the temperatures together. I divided the sum by 9, the number of temperature readings, to determine the mean noon temperature.

The mean noon temperature at Edmonton International Airport on January 1 is about −10.6 °C.

This answer seems reasonable since there were four temperatures above this amount and five temperatures below it.

EXAMPLE 2 | Solving a problem involving rational numbers

a) Use the numbers 1 and -5 in the blanks so that $\frac{\bullet}{\blacklozenge} \times 2\frac{3}{4}$ has the least possible value.

b) Use the numbers 1 and 2 in the blanks so that $\frac{-\bullet}{5} \div \blacksquare\frac{3}{4}$ has the greatest possible value.

Jia-Wen's Solution

a) The missing number is either

$$\frac{1}{-5} = -\frac{1}{5}$$

or $\frac{-5}{1} = -\frac{5}{1}$

The product had to be negative since the fraction involved a positive and negative value.

$\frac{1}{5} < \frac{5}{1}$, so $-\frac{1}{5} > -\frac{5}{1}$

$-\frac{5}{1} \times 2\frac{3}{4} < -\frac{1}{5} \times 2\frac{3}{4}$

If you multiply a negative number by a positive number, the answer is less if the negative number is less.

$$\frac{-5}{1} \times 2\frac{3}{4} = -5 \times \frac{11}{4}$$
$$= -\frac{55}{4}$$

I checked by multiplying.
I was right, since $-\frac{55}{4}$ is almost -14 and $-\frac{11}{20}$ is not even -1.

$$\frac{1}{-5} \times 2\frac{3}{4} = -\frac{1}{5} \times \frac{11}{4}$$
$$= -\frac{11}{20}$$

$-\frac{5}{1} \times 2\frac{3}{4}$ gives the least product.

b)
$$\frac{-1}{5} \div 2\frac{3}{4} = -\frac{1}{5} \div \frac{11}{4}$$
$$= -\frac{1}{5} \times \frac{4}{11}$$
$$= -\frac{4}{55}$$
$$\frac{-2}{5} \div 1\frac{3}{4} = -\frac{2}{5} \div \frac{7}{4}$$
$$= -\frac{2}{5} \times \frac{4}{7}$$
$$= -\frac{8}{35}$$

The only possibilities are $-\frac{1}{5} \div 2\frac{3}{4}$ or $-\frac{2}{5} \div 1\frac{3}{4}$. To divide, I converted the mixed number and multiplied by the reciprocal.
$-\frac{4}{55}$ is about $-\frac{1}{12}$.
$-\frac{8}{35}$ is about $-\frac{1}{4}$, which is less.

$-\frac{2}{5} \div 1\frac{3}{4}$ gives the greatest quotient.

In Summary

Key Ideas

- Multiplying and dividing rational numbers in decimal form combines the rules for multiplying and dividing positive decimals with the rules for multiplying and dividing integers. For example,

$$(-3.2) \div 1.2 = -(3.2 \div 1.2)$$

- Multiplying and dividing rational numbers in the form of fractions combines the rules for multiplying and dividing positive fractions with the rules for multiplying and dividing integers. For example,

$$5\frac{3}{4} \times \left(-2\frac{1}{3}\right) = -\left(\frac{23}{4} \times \frac{7}{3}\right)$$

Need to Know

- You can divide rational numbers in the form of fractions by using a common denominator and dividing the numerators. For example,

$$-\frac{12}{25} \div \frac{3}{5} = -\frac{12}{25} \div \frac{15}{25}$$

- You can also divide by multiplying by the reciprocal. For example,

$$-\frac{12}{25} \div \frac{3}{5} = -\frac{12}{25} \times \frac{5}{3}$$

Checking

1. Evaluate.

a) $(-2)(9.5)$

b) $\frac{-4}{7} \times \frac{6}{-5}$

c) $(-8) \div (0.5)$

d) $\frac{2}{5} \div \left(-\frac{5}{8}\right)$

2. How much less is $\frac{-3}{4} \div \frac{5}{6}$ than $\frac{-3}{4} \times \frac{5}{6}$?

3. A water tank lost $\frac{1}{3}$ of its volume of water one day and then $\frac{1}{2}$ of what was left the next day. What rational number describes the volume of water after the second day as compared to the original volume?

Practising

4. Calculate.

a) $-\frac{2}{3} \times \frac{5}{8}$

b) $-\frac{2}{3} \times \frac{-5}{8}$

c) $\frac{2}{3} \times \frac{-8}{5}$

d) $-\frac{5}{8} \div \frac{2}{3}$

e) $\frac{-2}{3} \div \left(-\frac{5}{8}\right)$

f) $\frac{2}{3} \div \frac{5}{8}$

5. **Multiple choice.** Which expression is about $-\frac{1}{2}$?

A. $-\frac{2}{3} \times \frac{1}{8}$ **B.** $\frac{8}{9} \div \left(-\frac{1}{2}\right)$ **C.** $\frac{4}{5} \div \left(1\frac{1}{2}\right)$ **D.** $-\frac{2}{3} \div \frac{5}{4}$

6. **Multiple choice.** Without evaluating, determine which expressions have the same product as $\left(\frac{3}{4}\right)\left(\frac{5}{8}\right)$.

W: $\left(\frac{-3}{-4}\right)\left(\frac{5}{8}\right)$ X: $-\left(\frac{3}{4}\right)\left(-\frac{5}{8}\right)$ Y: $\left(-\frac{3}{8}\right)\left(-\frac{5}{4}\right)$ Z: $\left(\frac{-3}{4}\right)\left(\frac{5}{-8}\right)$

A. X and Y **C.** X, Y, and Z

B. X and Z **D.** all of these expressions

7. Use the numbers -1, -3, and 8 in the blanks so that $\frac{\square}{\square} \times \square\frac{2}{3}$ has

a) the least possible value

b) the greatest possible value

Year	Temperature (°C)
2002	−20.4
2003	−7.6
2004	−15.8
2005	−9.3
2006	−10.5
2007	−16.3

Source: Environment Canada

8. Consider the numbers -4.2, -1.3, -8.4, and 7.3.

a) Which two have a product of 35.28?

b) Which two have a quotient of about -1.75?

9. The temperatures at Fort Nelson, BC, at 5:00 a.m. on December 25 from 2002 to 2007 are shown in the table. Determine the mean temperature at 5:00 a.m. on December 25 for these years.

10. Calculate. Show your work.

a) $\left(\frac{5}{-12}\right)\left(\frac{-8}{15}\right)$ **c)** $\frac{15}{16} \div \left(-1\frac{1}{24}\right)$ **e)** $(-3.2) \div (-8.4)$

b) $\left(3\frac{6}{7}\right)\left(-8\frac{1}{3}\right)$ **d)** $-4\frac{2}{3} \div \frac{7}{12}$ **f)** $7.2 \div (-0.6)$

11. A formula to convert temperatures between degrees Fahrenheit and degrees Celsius is $C = \frac{5}{9}(F - 32)$. Use this formula to convert the following.

a) Miami, Florida's record high of 98 °F to degrees Celsius

b) Anchorage, Alaska's record low of −37 °F to degrees Celsius

c) 0 °C to degrees Fahrenheit

12. Two tanks hold the same amount of water. Tank 1 loses $\frac{2}{3}$ of its volume. Tank 2 gains $\frac{1}{4}$ of its volume. What is the final ratio of water volume, comparing tank 1 to tank 2?

13. An investment loses $\frac{1}{2}$ of its value and then loses another $\frac{2}{3}$ of the new value.

a) What fraction of its original value is the final value?

b) Can you multiply $-\frac{1}{2} \times \left(-\frac{2}{3}\right)$ to calculate the answer to part a)? Explain.

14. A pail of water has been sitting for a while, and $\frac{1}{8}$ of the water has evaporated.

a) What could $\frac{3}{4} \times \left(-\frac{1}{8}\right)$ describe about this situation?

b) What could $-\frac{1}{8} \div \left(-\frac{1}{4}\right)$ describe about this situation?

15. The product of two rationals is $-\frac{12}{25}$. What might their quotient be?

16. The product of $-\frac{3}{4}$ and two other rationals is $\frac{1}{4}$. The quotient of the two other rationals is $-\frac{3}{4}$.

$$-\frac{3}{4} \times \frac{\bullet}{\blacklozenge} \times \frac{\blacktriangle}{\blacksquare} = \frac{1}{4}$$

$$\frac{\bullet}{\blacklozenge} \div \frac{\blacktriangle}{\blacksquare} = -\frac{3}{4}$$

a) How do you know that one unknown rational is positive and one is negative?

b) What could the unknown rationals be?

17. Evaluate each expression for the given values. Use a calculator.

a) $x - 2y$ when $x = -9.78$ and $y = 3.2$

b) $(x + y)(x - y)$ when $x = 2.5$ and $y = -7.8$

c) $x(x + y)$ when $x = -2\frac{1}{2}$ and $y = 3\frac{3}{4}$

d) $\frac{x}{y} + \frac{y}{x}$ when $x = -1\frac{1}{2}$ and $y = 2\frac{1}{4}$

Closing

18. Create a brief "instruction manual" to help someone with the rules for multiplying and dividing rational numbers.

Extending

19. Calculate each product.

a) $-9 \times 0.2222...$ **b)** $-99 \times 0.232\,323...$

20. The product of a positive and a negative rational number is 2 greater than their sum. What could they be?

21. The width of a rectangle is $\frac{1}{4}$ of the length. If you increase the width by 12 m and double the length, you obtain a perimeter of 60 m. Determine the dimensions of the original rectangle.