1.3 Adding and Subtracting Rational Numbers

YOU WILL NEED

- a calculator with fraction capability

GOAL

Solve problems that involve adding and subtracting rational numbers.

LEARN ABOUT the Math

Jia-Wen's father started working for a new company and was given shares in the company. Jia-Wen paid attention to how those share prices were changing.

Changes in share prices are sometimes reported using rational numbers. A positive number describes an increase in price and a negative number describes a decrease.

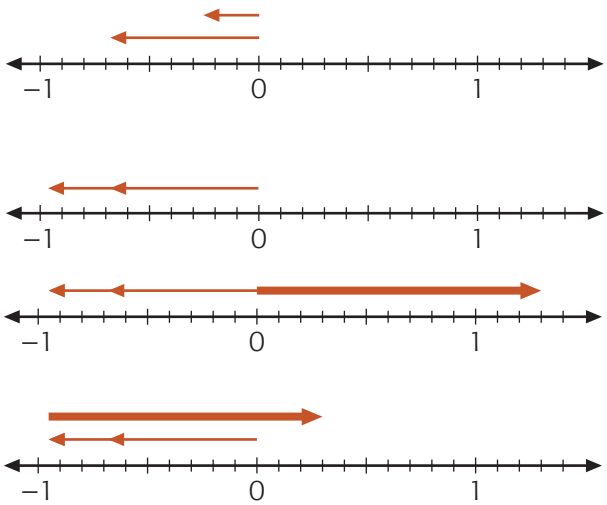
The dollar value of a single share in Jia-Wen's father's company changed as follows:

Week 1: -0.68 Week 2: -0.25 Week 3: $+1.34$

? By how much did the increase in week 3 make up for the losses in weeks 1 and 2?

EXAMPLE 1 Adding and subtracting rationals

Jia-Wen's Solution: Representing sums and differences with a number line



I used a number line to estimate the total change. I modelled each loss by going to the left. Then, I added the second loss to the first one.

I drew an arrow 1.3 units long and moved it to begin at the end of the second loss.

I can tell the gain was greater than the combined loss since the positive arrow is longer than the two negative ones together.

$0.68 + 0.25 = 0.93$
So, the combined loss is
$-0.68 + (-0.25) = -0.93$

To see exactly how much more, I compared the total loss to the gain.

The positive gain is 1.34. The difference is
$1.34 - 0.93 = 0.41$
The gain is 0.41 more than the loss.

I compared the length of the gain arrow to the combined lengths of the loss arrows.

Rachel's Solution: Using reasoning to add and subtract

$$\begin{aligned}(-0.68) + (-0.25) &= -(0.68 + 0.25)\\ &= -(0.93)\\ &= -0.93\end{aligned}$$

I figured that if I am adding two negative numbers, the sum would be negative, just as with integers.

$$\begin{aligned}-0.93 + 1.34 &= 1.34 - 0.93\\ &= 0.41\end{aligned}$$

To add the gain of 1.34, I could subtract 0.93 from 1.34.

The final increase overcame the total loss by 0.41.

Sam's Solution: Using the zero principle

$(-0.68) + (-0.25) + 1.34 = x$

I decided I was adding the two negative numbers and one positive number. I decided to use the zero principle.

$$\begin{aligned}[-0.68 + 0.68] + [-0.25 + 0.25] + 1.34 &= x + 0.68 + 0.25\end{aligned}$$

I added the same amounts to both sides of the equation so that I could use the zero principle. I added 0.68 + 0.25.

$$\begin{aligned}0 + 0 + 1.34 &= x + 0.93\\ 1.34 &= x + 0.93\\ 1.34 - 0.93 &= x\\ 0.41 &= x\end{aligned}$$

I know that a number plus its opposite is 0.

Reflecting

A. How were the three solutions alike?

B. Why might one person think of the problem as an addition and another person think of it as a subtraction?

C. How is adding rationals like adding integers?

WORK WITH the Math

EXAMPLE 2 | Estimating the sum and difference of rational numbers

A temperature changed over the course of 3 h as follows:

Hour 1: $+2.3$ °C
Hour 2: -4.5 °C
Hour 3: -2.7 °C

About how much higher or lower than the original temperature was the final temperature?

Thomas's Solution

-2.7 is almost the opposite of 2.3, so those two temperatures add to a value close to 0 °C.

I noticed that the temperatures in hour 1 and hour 3 were close to opposites.

The final temperature was about 4 °C or 5 °C below the original temperature.

I realized I just needed to use the decrease in hour 2 to estimate the total decrease.

EXAMPLE 3 | Adding rationals in fraction form

Calculate $3\frac{1}{4} + \frac{-7}{3}$.

Larissa's Solution

$3\frac{1}{4} + \frac{-7}{3}$ is about $3 + (-2) = 1$.

I estimated first. $3\frac{1}{4}$ is a little more than 3 and $\frac{-7}{3} = -2\frac{1}{3}$ is about -2. I used $3 + (-2)$ to estimate.

$3\frac{1}{4} = \frac{13}{4} = \frac{39}{12}$
$\frac{-7}{3} = -\frac{7}{3} = -\frac{28}{12}$

I wrote $3\frac{1}{4}$ as $\frac{13}{4}$. Then, I wrote each rational using equivalent fractions with the same denominator, 12.

$3\frac{1}{4} + \frac{-7}{3} = \frac{39}{12} + \left(-\frac{28}{12}\right)$
$= \frac{11}{12}$

I added the rationals by adding the numerators.

$\frac{11}{12}$ is close to 1, so my answer is reasonable.

I compared my result to my estimate to check.

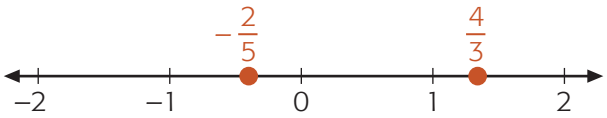
EXAMPLE 4 | Subtracting rationals in fraction form

Calculate $1\frac{1}{3} - \frac{2}{-5}$.

David's Solution: Using a number line to visualize

$1\frac{1}{3} - \frac{2}{-5}$ is the distance from $\frac{2}{-5} = -\frac{2}{5}$ to $1\frac{1}{3} = \frac{4}{3}$ on a number line.

I know that to calculate a difference, I can think about the distance from the second value to the first one.

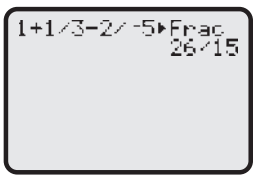


$\frac{2}{5} = \frac{6}{15}$ and $\frac{4}{3} = \frac{20}{15}$

I added $\frac{2}{5}$ to get from $-\frac{2}{5}$ to 0 and $\frac{4}{3}$ to get from 0 to $\frac{4}{3}$.

$\frac{6}{15} + \frac{20}{15} = \frac{26}{15}$ or $1\frac{11}{15}$

Thomas's Solution: Using a calculator



To subtract a negative, I used the – key for the subtraction and the (−) key to change the value of 5 to a negative.

$\frac{26}{15} = 1\frac{11}{15}$

In Summary

Key Ideas

- Adding and subtracting rational numbers in the form of decimals combines the rules for adding and subtracting positive decimals with the rules for adding and subtracting integers.

  For example, $-4.3 + 5.25 = 5.25 - 4.3 = 0.95$.

- Adding and subtracting rational numbers in the form of fractions combines the rules for adding and subtracting positive fractions with the rules for adding and subtracting integers.

  For example, $5\frac{3}{4} - \left(-2\frac{1}{3}\right) = 5\frac{3}{4} + 2\frac{1}{3}$.

Need to Know

- It is useful to estimate sums and differences to verify calculations of sums and differences.
- You can visualize a number line and use a combination of locations and distances to estimate and calculate sums and differences of rationals.

Checking

1. Evaluate.

a) $-4.2 + (-3.8)$

b) $\frac{7}{8} + \left(-\frac{2}{3}\right)$

c) $2\frac{1}{5} + \left(-\frac{1}{2}\right)$

d) $2.5 - 5.6$

e) $-\frac{4}{3} - \left(-\frac{3}{4}\right)$

f) $-3\frac{4}{5} - 1\frac{2}{3}$

2. You lose \$1.20 on each share you own and then gain back \$0.65. Write the total loss on each share as a rational number.

Practising

3. Estimate the sums or differences. Explain your thinking.

a) $3.64 - 72.9$

b) $-12.2 - (-18.9)$

c) $-9.37 - 5.93$

d) $0.47 - (-21.6)$

e) $3.42 - (-5.6) + 11.3$

f) $-5.1 + (-5.82) + 5.01$

4. Calculate exact answers for question 3.

5. **Multiple choice.** Which sum or difference is about $+16$?

A. $-2.3 - 18.4$

B. $14.1 + (-2.1)$

C. $-4.1 - (-19.8)$

D. $23.98 + (-8.9)$

6. **Multiple choice.** Yaroslav takes $\frac{3}{4}$ h to cut his family's front lawn and $1\frac{1}{3}$ h to cut the back lawn. How much longer does it take Yaroslav to cut the back lawn than the front?

A. $1\frac{1}{2}$ h

B. $1\frac{1}{3}$ h

C. 35 min

D. 45 min

7. Consider these numbers: -4.2, -8.94, -5.362, $+9.4$, $+1.205$
Which two numbers have

a) a sum of 5.2?

b) a difference of 6.567?

c) a sum of 4.038?

d) a difference of -3.578?

8. Determine the missing digits for each. Use a calculator to help you.

a) $-3.5\square 2 + \square.42\square = 1.846$

b) $-1\square.382 - (4.17\square) + 8.\square 3 = -7.\square 27$

c) $-2.45\square - (-5.\square 63) = \square.705$

d) $-5.1\square - (-\square.8) - 7.\square = -9.21$

9. **a)** How could using the zero principle help you add $3.4 + (-8.9)$?

b) Why would the zero principle not help you add $-3.4 + (-8.9)$? What other strategy could you use instead?

10. Calculate. Show your work.

a) $-\frac{3}{8} + 1\frac{3}{4}$

b) $-5\frac{1}{2} + 2\frac{2}{3}$

c) $\frac{6}{5} - \frac{3}{2}$

d) $1\frac{3}{4} + \left(-3\frac{2}{5}\right)$

e) $-3\frac{2}{3} - 4\frac{1}{5}$

f) $\frac{5}{-8} - \left(-\frac{11}{3}\right)$

11. The daily changes in price for a share during a week were $-\$2.78$, $-\$5.45$, \$0.38, \$1.38, and \$2.12. The price of the share was \$58.22 at the start of the week. What was the price at the end of the week?

12. How do you know that $-2.3 - \left(-3\frac{1}{4}\right)$ is

a) greater than $-2.3 + \left(-3\frac{1}{4}\right)$? **b)** about 1?

13. Determine the value that makes each equation true.

a) $-1\frac{3}{4} + \square = 1$ **b)** $-1\frac{3}{4} - \square = 1$

14. James finished the Manitoba Marathon in a time of 3:57:53.3 (hours: minutes: seconds). The winner of the marathon finished in a time of 2:25:55.6. Determine how much longer James took to complete the marathon than the winner did.

15. Evaluate each expression for the given values.

a) $x - y$ when $x = -4.1$ and $y = -3.2$

b) $x + y + z$ when $x = 2.5$, $y = -7.8$, and $z = -4.1$

c) $x - y$ when $x = -2\frac{1}{2}$ and $y = -3\frac{3}{4}$

d) $x + y$ when $x = -1\frac{1}{2}$ and $y = 2\frac{1}{4}$

16. To recreate the work of the voyageurs during the fur trade, a relay race was held on the Red River near St-Boniface, MB. Participants canoed to specific points to find a message like those at right, which led them to a fur cache. What rational number operations would you use to determine each of the following?

a) the distance of the last leg **b)** the total distance paddled

Message 1: 1.5 km south
Message 2: 0.68 km north
Message 3: 2.3 km south
Message 4: north to the starting point

17. List two rational numbers a and b that are not integers and that would make each statement true.

a) $a + b$ is negative, but $a - b$ is positive.

b) $a + b$ is positive, but $a - b$ is negative.

c) $a + b$ and $a - b$ are both negative.

18. Describe a real-world problem where you might calculate $-3.2 - (-4.5)$. Solve your problem.

Closing

19. Describe a strategy for calculating the sum and a strategy for calculating the difference of -3.4 and $+5.005$.

Extending

20. The sum of two rational numbers is $\frac{23}{40}$. Their difference is $-1\frac{3}{40}$. What are the numbers?

21. The sum of two rationals is 17.4 less than the difference. What could the rationals be?