1.2 Comparing and Ordering Rational Numbers

GOAL

Order a set of rational numbers.

LEARN ABOUT the Math

Larissa described going east from her house as moving in a positive direction and going west as moving in a negative direction.

On five different days, she described how far she was from home:

-3.7 km -5.2 km -5.3 km 5.1 km -1.4 km

? On which day was Larissa farthest from home?

A. Draw a number line from -10 to 10. Estimate and mark the position of each of Larissa's locations on the number line if her home is at 0.

B. List the location numbers from least to greatest.

C. Which position was farthest from home?

Reflecting

D. How does your answer to part C show that just because a number is farthest from 0 in a list of numbers, it might not be the greatest?

E. How could you have looked at the location numbers and ordered them without marking them on the number line?

WORK WITH the Math

EXAMPLE 1 | Ordering rationals in fraction and decimal form

Order these numbers from least to greatest.

$-3.4 \quad -5\frac{1}{2} \quad 12.1 \quad 15\frac{2}{3} \quad -5\frac{3}{4} \quad \frac{8}{3}$

David's Solution

The three greatest numbers are the positives.

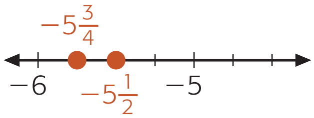
$\frac{8}{3} = 2\frac{2}{3} < 12.1 < 15\frac{2}{3}$

First I wrote $\frac{8}{3}$ as a mixed number. It's between 2 and 3, so it's less than 12.1 and $15\frac{2}{3}$.

Since these are the only positives, I know they are the greatest.

$-3.4 > -5\frac{1}{2}$ and $-5\frac{3}{4}$

I know that $-3.4 > -4$ and that the other two negatives are less than -4, so -3.4 is the greatest negative.



I divided the part of the number line between -5 and -6 into 4 equal sections to place the last two numbers. $-5\frac{3}{4}$ is farther to the left, so it's least.

The order of the numbers from least to greatest is

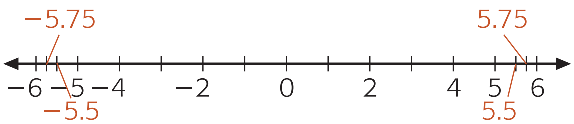
$-5\frac{3}{4} \quad -5\frac{1}{2} \quad -3.4 \quad \frac{8}{3} \quad 12.1 \quad 15\frac{2}{3}$

Larissa's Solution

$$-5\frac{1}{2} = -5.5$$
$$-5\frac{3}{4} = -5.75$$
$$\frac{8}{3} = 1\frac{2}{3} \doteq 1.67$$

I know that $15\frac{2}{3}$ is the greatest. I decided to write the other numbers in fractional form as decimals, since I find decimals easier to work with.

The five numbers that are less than $15\frac{2}{3}$ are -3.4, -5.5, 12.1, -5.75, and 1.67.



I know that -3.4 is greater than the other negative values and that 1.67 and 12.1 are greater than all the other decimals.

I know that $-5.75 < -5.5$ since the opposite of 5.5 is closer to 0 than the opposite of 5.75.

I thought of the numbers on a number line.

The order of all the numbers except $15\frac{2}{3}$, as decimals, is -5.75, -5.5, -3.4, 1.67, 12.1.

The order of the numbers from least to greatest is

$-5\frac{3}{4} \quad -5\frac{1}{2} \quad -3.4 \quad \frac{8}{3} \quad 12.1 \quad 15\frac{2}{3}$

EXAMPLE 2 Comparing rationals that describe situations

Order these rational numbers from least to greatest.

$-4\frac{1}{2}, 8\frac{1}{4}, -2\frac{1}{4}, -\frac{5}{8}, -2\frac{3}{4}, -\frac{5}{10}$, and $5\frac{3}{4}$

Rachel's Solution

$5\frac{3}{4} < 6$ and $8\frac{1}{4} > 6$, so $8\frac{1}{4}$ is greater than $5\frac{3}{4}$.

I know that $5 < 8$, but $\frac{3}{4} > \frac{1}{4}$. To be sure which was least, I compared both numbers to a benchmark of 6.

These are the two greatest values.

I know that negative values are less than positive ones, so the five negative values are less than $5\frac{3}{4}$ and $8\frac{1}{4}$.

$-4\frac{1}{2} < -4$

$-2\frac{1}{4} > -3$, which is greater than -4.

$-2\frac{3}{4} > -3$, which is greater than -4.

So, $-4\frac{1}{2} < -2\frac{1}{4}$ and $-4\frac{1}{2} < -2\frac{3}{4}$.

I compared three of the negatives to a benchmark of -4; one was to the left and two were to the right of -4.

$\frac{1}{4} < \frac{3}{4}$, since $1 < 3$.

When the denominators of two fractions are the same, you can compare numerators.

So, $-\frac{3}{4} < -\frac{1}{4}$

If a positive fraction is closer to 0 than another, its opposite is also closer to 0, and so it is greater.

$-2\frac{3}{4} < -2\frac{1}{4}$

$-2\frac{3}{4}$ and $-2\frac{1}{4}$ are each exactly 2 to the left of $-\frac{3}{4}$ and $-\frac{1}{4}$, so $-2\frac{3}{4}$ is farther to the left of 0 than $-2\frac{1}{4}$.

$-\frac{5}{8}$ and $-\frac{5}{10}$ are both greater than $-2\frac{3}{4}$, $-2\frac{1}{4}$, and $-4\frac{1}{2}$.

$-\frac{5}{8}$ and $-\frac{5}{10}$ are both between 0 and -1, so they are greater than -1.

$\frac{5}{8} > \frac{5}{10}$, so $-\frac{5}{8} < -\frac{5}{10}$.

$\frac{5}{10}$ and $\frac{5}{8}$ have the same numerator, but 10 is a greater denominator. $\frac{5}{8}$ is farther to the right of zero than $\frac{5}{10}$, so $-\frac{5}{8}$ is farther to the left of zero than $-\frac{5}{10}$.

From least to greatest, the rational numbers are

$-4\frac{1}{2}, -2\frac{3}{4}, -2\frac{1}{4}, -\frac{5}{8}, -\frac{5}{10}, 5\frac{3}{4}$, and $8\frac{1}{4}$.

I realized I could have just compared the integer parts for the rational numbers with different integer parts: $-4 < -2 < 0 < 5 < 8$.

In Summary

Key Ideas

- Rationals can be compared in the same ways as integers and fractions.
- Negatives are always less than positives. A negative farther from 0 is always less than a negative closer to 0.

Need to Know

- To compare rationals in fraction form, it helps to use mixed number representations, equivalent fractions with common denominators or common numerators, or benchmarks. For example,

$$-3\tfrac{1}{2} > -4\tfrac{1}{3} \text{ since } -3 > -4$$

$$-\tfrac{3}{5} > -\tfrac{7}{8} \text{ since } -\tfrac{24}{40} > -\tfrac{35}{40} \text{ or since } -\tfrac{21}{35} > -\tfrac{21}{24}$$

$$-\tfrac{2}{3} < -\tfrac{1}{5} \text{ since } -\tfrac{2}{3} < -\tfrac{1}{2} \text{ and } -\tfrac{1}{5} > -\tfrac{1}{2}$$

- You can also express all the rational numbers as decimals and compare them in decimal form.

Checking

1. List three rationals between $-5\frac{2}{3}$ and $-4\frac{2}{3}$.

2. Explain why $-3\frac{3}{4} < -3\frac{1}{8}$.

3. You and your friends are standing at a point called 0. You use positive numbers to describe going east and negative numbers to describe going west. One friend moves to $+3.2$. Another moves to -2.3. A third moves to -0.9. Which of your friends moved the least distance? In what direction?

Practising

4. Use $>$, $<$, or $=$ to make each statement true. Explain how you know parts b) and d) are true.

a) $0 \blacksquare -0.5$

b) $-4.3 \blacksquare -3.4$

c) $-1\frac{2}{5} \blacksquare 1\frac{2}{5}$

d) $-4\frac{1}{2} \blacksquare -\frac{9}{2}$

e) $5.6 \blacksquare 5\frac{3}{5}$

f) $-2\frac{3}{10} \blacksquare 2.\overline{3}$

5. Replace the $\blacksquare$ with a value to make each statement true.

a) $-2.3 < -\blacksquare.4$

b) $-1.\blacksquare < -\blacksquare.8$

c) $-0.\blacksquare < 0.\blacksquare$

d) $1.9 < \blacksquare.4$

e) $-1.\blacksquare > -1.\blacksquare$

f) $\blacksquare.8 > 4.\blacksquare$

6. Order each from least to greatest.

a) $-12.2, -14\frac{1}{2}, 3\frac{1}{4}, -4\frac{2}{3}$

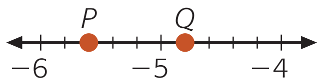
b) $-8.4, -10.2, -10.3, 8.8, 8\frac{2}{3}$

c) $\frac{3}{8}, -\frac{3}{8}, \frac{3}{15}, \frac{-3}{15}$

d) $14.2, -2.9, -2\frac{3}{4}, 10.1$

7. Draw X and Y on a number line to make both statements true.
X is between -2 and -3 but greater than -2.8.
Y is greater than X but less than -2.2.

8. **Multiple choice.** Which number sentence describes how P compares to Q?



A. $-5.6 < -4.8$

B. $-5.3 < -4.4$

C. $-5.6 > -4.8$

D. $-5.3 > -4.4$

9. **Multiple choice.** Which correctly orders $+8.1, -4.3, -4\frac{1}{3}, +0.9$, and $-\frac{24}{5}$ from least to greatest?

A. $-4.3, -4\frac{1}{3}, -\frac{24}{5}, 0.9, 8.1$

B. $-4\frac{1}{3}, -4.3, -\frac{24}{5}, 0.9, 8.1$

C. $-\frac{24}{5}, -4\frac{1}{3}, -4.3, 0.9, 8.1$

D. $-\frac{24}{5}, -4\frac{1}{3}, -4.3, 8.1, 0.9$

10. Earth scientists sometimes use negative values to describe the time before a major earthquake. Suppose a scientist observed a tremor at -12.2 s and another tremor at -10.4 s. Which tremor occurred first? How do you know?

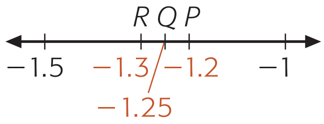
11. Why would you probably use a different strategy to compare -4.3 and $-4\frac{1}{3}$ than to compare $+8.1$ and -4.3?

12. **a)** Describe three different strategies you might use to order these rational numbers.

$$-\frac{11}{2}, \quad -7.5, \quad -\frac{7}{5}, \quad -1\frac{1}{2}, \quad -2\frac{1}{2}, \quad \frac{5}{8}, \quad \frac{1}{100}$$

b) Use your strategies to order the numbers from least to greatest.

13. Point Q, -1.25, is between point P at -1.2 and point R at -1.3.



a) Copy the number line. Draw a point S between P and Q. Label it with a value.
b) Draw a point T between P and S. Label it with a value.
c) Repeat two more times, each time drawing a new point between P and the last point and labelling it with a value.
d) How can you be sure there is always another rational number between two given ones? Use an example to explain.
e) What does this tell you about how many rational numbers there must be?

14. If a and b are positive numbers and $a < b$, how do $-a$ and $-b$ compare? Explain why.

15. Rational numbers can be written in either decimal form or fraction form. Which form do you find easier to use when ordering rational numbers? Use examples to justify your decision.

Closing

16. Agree or disagree with the following statement and explain your reasons. "If you can order integers and you can order fractions, you have all the skills needed to order rational numbers."

Extending

17. Order these from least to greatest: $-0.242\,424\ldots$, $-\frac{1}{4}$, $-\frac{24}{100}$, $-\frac{24}{98}$

18. The same digit is substituted into each blank. The order from least to greatest is

$$-\blacksquare\frac{3}{5},\ -7\frac{1}{\blacksquare},\ -4\frac{\blacksquare}{11},\ -4\frac{2}{\blacksquare}.$$

What could the digit be? Explain.

19. Let x represent a value that could be anywhere between -2.4 and -3.8. Let y represent a value that could be anywhere between -4 and -3.5. About what fraction of the time will it be true that $x < y$?