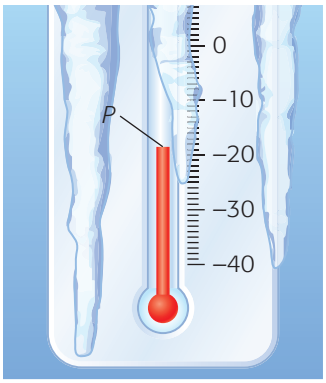
1.1 Interpreting Rational Numbers

GOAL

Relate rational numbers to fractions and integers.

LEARN ABOUT the Math

Rachel looked at the thermometer outside. The temperature was between −18 °C and −19 °C.



rational number

a number that can be expressed as the quotient of two integers, where the divisor is not 0; it can be written in fraction, mixed-number, or decimal form or as an integer; e.g., 3.25, −5.8, $\frac{2}{3}$, −2, $-1\frac{1}{4}$

opposites

two numbers with opposite signs that are the same distance from 0; e.g., +2 and −2 or +0.5 and −0.5

Communication *Tip*

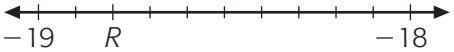
Sometimes rational numbers are called rationals. Rational numbers like −0.6 and $\frac{5}{8}$ are read as "negative six tenths" and "positive five eighths" or just "five eighths."

? What might the temperature be?

A. What is the value of the **rational number** located at P? Explain.

B. What other form of that number could you use to describe P?

C. What is the value of the rational number located at R? Explain.



D. Why are the values at P and R both possible solutions to Rachel's problem?

E. List three other possible solutions to the problem. Write one in decimal form, one as the **opposite** of an improper fraction, and one as the opposite of a mixed number.

Reflecting

F. How could thinking about the opposites of −18 and −19 help you solve Rachel's problem?

G. How do you know that between any two rationals there are always many other rationals?

H. How do you know that any integer or fraction is also a rational number?

WORK WITH the Math

EXAMPLE 1 | Recognizing equivalent rational numbers

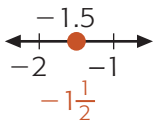
Which of the following represent the same rational number?

$$\frac{-3}{2},\quad \frac{3}{-2},\quad \frac{-6}{-4},\quad -\frac{6}{4},\quad -1\frac{1}{2},\quad -1.5$$

Thomas's Solution

$\frac{-3}{2}$, $\frac{3}{-2}$, $\frac{-6}{-4}$, $-\frac{6}{4}$, $-1\frac{1}{2}$, -1.5
A B C D E F

I will label the numbers to make sure that I use them all.



The number halfway between -1 and -2 should be $-1\frac{1}{2}$, but it's also -1.5 since $\frac{1}{2} = 0.5$.

$-1\frac{1}{2} = -1.5$ so $E = F$

$-\frac{6}{4} = -\frac{3}{2}$

$-\frac{3}{2} = -1\frac{1}{2}$

Since $\frac{6}{4} = \frac{3}{2}$, then $-\frac{6}{4} = -\frac{3}{2}$.
Since $\frac{3}{2} = 1\frac{1}{2}$, then $-\frac{3}{2} = -1\frac{1}{2}$.

$-1\frac{1}{2} = -\frac{6}{4}$ so $E = D$

C is different from D, E, and F.

I know that $\frac{-6}{-4} = (-6) \div (-4)$. When you divide two negatives, the result is positive, so this cannot be the same as -1.5.

$\frac{-3}{2} = (-3) \div 2$
$= -1.5$
$A = F$

The only ones left to think about are $\frac{-3}{2}$ and $\frac{3}{-2}$.
If there is a loss of \$3 that 2 people share, each loses \$1.50, so it makes sense that $\frac{-3}{2} = -1.5$.

$\frac{3}{-2} = 3 \div (-2)$
$= -1.5$
$B = F$

To figure out the quotient, I think about what number to multiply (-2) by to get $+3$. It has to be negative to get a positive product. It has to be -1.5 since $1.5 \times 2 = 3$.

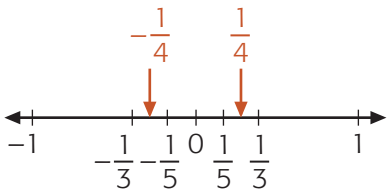
Since $A = B = D = E = F$, the equivalent rational numbers are $\frac{-3}{2}$, $\frac{3}{-2}$, $-\frac{6}{4}$, $-1\frac{1}{2}$, and -1.5.

All five of these numbers are located at the same place on a number line.

EXAMPLE 2 | Locating a rational number between two other rational numbers

List five different rational numbers between $-\frac{1}{5}$ and $-\frac{1}{3}$.

Larissa's Solution



It helps to place $-\frac{1}{5}$ and $-\frac{1}{3}$ on a number line first. I did this by placing their opposites on the number line, and then reflecting across zero.
Opposites must be the same distance from 0. Since $\frac{1}{4}$ is between $\frac{1}{3}$ and $\frac{1}{5}$, I know that $-\frac{1}{4}$ must be between $-\frac{1}{5}$ and $-\frac{1}{3}$.

$-\frac{1}{3} = -0.3333\ldots$
$-\frac{1}{5} = -0.2$

I changed the fractions to decimals to get other in-between rational numbers.

Since 0.3 and 0.28 are between 0.2 and 0.333..., then -0.3 and -0.28 must be between $-\frac{1}{5}$ and $-\frac{1}{3}$.

$-\frac{1}{5} = -\frac{6}{30}$ and $-\frac{1}{3} = -\frac{10}{30}$,

I used equivalent fractions.

so $-\frac{7}{30}$ and $-\frac{8}{30}$ are between them.
Five rational numbers between $-\frac{1}{5}$ and $-\frac{1}{3}$ are $-\frac{1}{4}$, -0.3, -0.28, $-\frac{7}{30}$, and $-\frac{8}{30}$.

EXAMPLE 3 | Using a rational number to describe a situation

Name three situations that you might describe using the rational number -3.4.

Sam's Solution

-3.4 could be a temperature between -3 °C and -4 °C.

-3.4 m could be the number of metres below sea level that a scuba diver is.

-3.4 could be the number of seconds before the launch of a spacecraft, if you call the launch of the spacecraft time 0.

I know that you can have negative temperatures, negative distances, and negative times (times before an event), so I'll use those ideas.

In Summary

Key Idea

- Rational numbers include integers, fractions, their decimal equivalents, and their opposites.

Need to Know

- Rational numbers can be positive, negative, or zero.
- Every integer is a rational number because it can be written as the quotient of two integers.

  For example, some ways -5 can be written are $\frac{-5}{1}$, $\frac{5}{-1}$, and $\frac{-10}{2}$.
- Just as with fractions, there are many ways to write the same rational number.

  For example, $-2.5 = \frac{-5}{2} = -\frac{5}{2} = \frac{5}{-2} = -2.50 = -\frac{10}{4}$.
- Every rational number (except 0) has an opposite. For example, $-2\frac{3}{4}$ and $2\frac{3}{4}$ are opposites, since they are both the same distance from 0 on a number line.

Checking

1. Write the following rational numbers as quotients of two integers.

a) -0.5 **b)** $-2\frac{1}{4}$ **c)** $-5\frac{6}{7}$ **d)** 7.2

2. Write the following rational numbers in decimal form.

a) $-\frac{1}{4}$ **b)** $-2\frac{3}{8}$ **c)** $-7\frac{5}{6}$ **d)** $4\frac{3}{10}$

3. List three rational numbers between each pair.

a) $-\frac{2}{3}$ and $-\frac{4}{5}$ **b)** $-\frac{1}{2}$ and $\frac{1}{4}$ **c)** 0.6 and $1\frac{1}{8}$

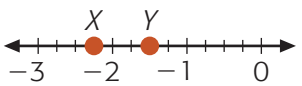
Reading Strategy

Visualizing

Use number lines to visualize your answers to question 3.

Practising

4. Multiple choice. Which values describe the positions of X and Y?



A. -2.2 and -1.5 **C.** $-2\frac{3}{4}$ and $-1\frac{1}{2}$

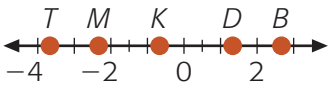
B. -2.1 and -1.2 **D.** $\frac{-9}{4}$ and $\frac{3}{-2}$

5. Multiple choice. Which of these rational numbers are equivalent?

X: -4.2 Y: -2.4 Z: $-\frac{42}{10}$ W: $\frac{24}{10}$

A. X, Z, and W **B.** X and Z **C.** X and W **D.** X and Y

6. Identify the values represented by B, D, K, M, and T as quotients of two integers.



7. Write each as the quotient of two rational numbers.

a) 5.1 **b)** $-4\frac{1}{5}$ **c)** -3.02 **d)** $9\frac{2}{3}$

8. Write each in decimal form.

a) $5\frac{2}{5}$ **b)** $-7\frac{2}{4}$ **c)** $-\frac{19}{3}$ **d)** $\frac{-13}{-5}$

9. Draw a number line. Mark integers from -10 to 2 on it. Estimate and mark the location of each of these on your number line.

a) $-\frac{5}{2}$ **c)** -7.2 **e)** $-3\frac{2}{3}$ **g)** -0.78

b) $1\frac{2}{3}$ **d)** -6.9 **f)** $\frac{2}{3}$ **h)** -8.4

10. **a)** Describe a real-life situation that involves the number -1.2.

b) Describe a real-life situation that involves the number $-\frac{2}{3}$.

11. Explain why $-\frac{3}{4} = \frac{-3}{4} = \frac{3}{-4}$.

12. **a)** Name three fractions between $\frac{2}{8}$ and $\frac{3}{8}$.

b) How would your answers in part a) help you name three rational numbers between $-\frac{2}{8}$ and $-\frac{3}{8}$?

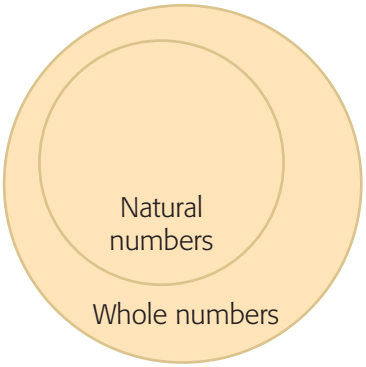
c) Are your answers in part a) rational numbers? Explain.

13. Agree or disagree with each. Explain why.

a) The opposite of every mixed number can be written as a rational number in decimal form.

b) If one number is greater than another, so is its opposite.

14. The natural numbers are the numbers 1, 2, 3, 4,...
The whole numbers are the numbers 0, 1, 2, 3,...
Extend this Venn diagram to show the relationship between these sets of numbers: integers, whole numbers, natural numbers, and rational numbers.



Closing

15. If you were describing rational numbers to someone without just repeating the definition, what is the most important thing you could say to help them quickly understand what rational numbers are?

Extending

16. **a)** Explain why $\frac{5}{9}$ can be written as the repeating decimal 0.555... (or $0.\overline{5}$).

b) How would you write $-\frac{5}{9}$ as a decimal?

c) How would you write $-\frac{5}{90}$ as a repeating decimal?

d) How are your answers to parts b) and c) related?

17. Design an appropriate symbol for the term rational number. Explain why your symbol is appropriate.