Math 9
Chapter 9 Circle Geometry
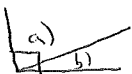
Notes
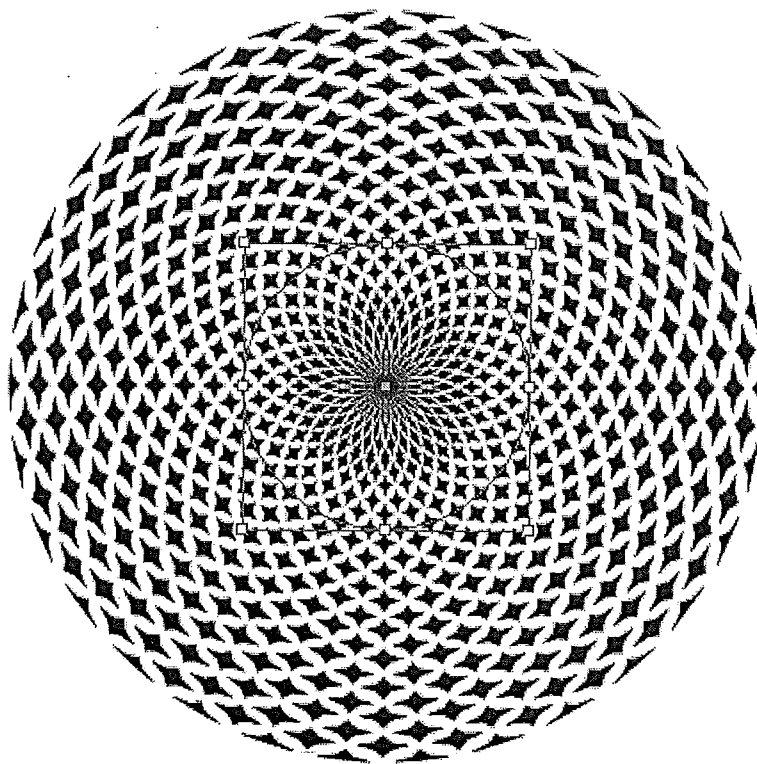
There are four different kinds of angles. Can you name them and draw them?
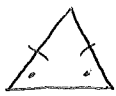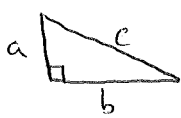
Name	Sketch	Explanation
Acute		less than 90°
Obtuse		greater than 90°
Right		90°
Straight		180°

There are five important rules that relate lines to each other
Name them, then sketch and explain them

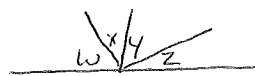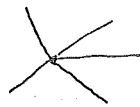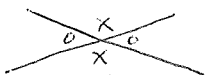| Name	Sketch	Explanation
Complementary		∠'s a & b add up to 90°
Supplementary		∠'s c and d add up to 180°
Angles on a line		angles w, x, y, z on a line add up to 180°
Angles @ a point		angles @ a point add up to 360°
Vertically opposite		vertically opposite ∠'s are equal

\

There are four different kinds of triangles. Can you identify them?

Recall → ∠ sum of a triangle is 180°

| Name	Sketch	Explanation
Scalene		-no equal sides -no ∠'s equal
Isosceles		-at least 2 sides equal -angles opposite the equal sides are equal
Equilateral		-3 sides equal -3 ∠'s are equal (60°)
Right	a, c, b	-1 right ∠ (90°) -hypotenuse is opposite right ∠ -$a^2 + b^2 = c^2$

Parallel Lines create some useful geometry rules, when they are intersected by a line called a TRANSVERSAL

a, y, x, b
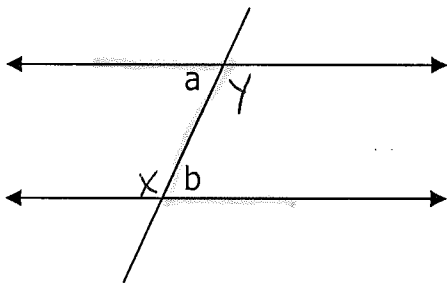
Angles a and b are a pair of ALTERNATE INTERIOR ANGLES. Can you find another pair? Look for the 'Z' shape $\angle a = \angle b$

∠x = ∠y

a, b, c, x, d, e, f, y
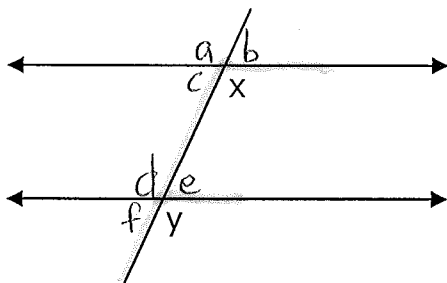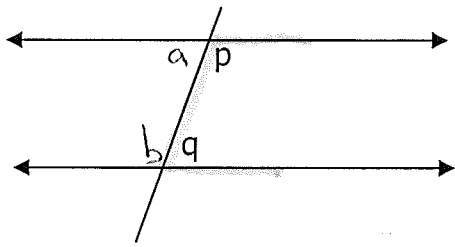
Angles x and y are a pair of equal CORRESPONDING ANGLES. How many other pairs are there? $\angle x = \angle y$
Look for the 'F' shape

∠a = ∠d
∠b = ∠e
∠c = ∠f

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Angles p and q are called
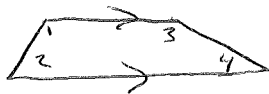CO-INTERIOR ANGLES.
They add to 180° ($\angle p + \angle q = 180°$)
Is there another pair? Look for the 'C'
shape

$$\angle a + \angle b = 180°$$

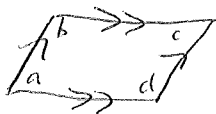What can you remember about Quadrilaterals? How many types are there?

→ $\angle$ sum of a quadrilateral is $360°$

Trapezoid

- 1 pair of parallel lines ($||$ → signifies parallel)
- $\angle 1 + \angle 2 = 180°$, $\angle 3 + \angle 4 = 180°$

Parallelogram

- opposite sides are equal and parallel
- opposite $\angle$'s are equal ($\angle a = \angle c$, $\angle b = \angle d$)
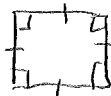- consecutive $\angle$'s equal $180°$

Rectangle

- opposite sides are equal & parallel
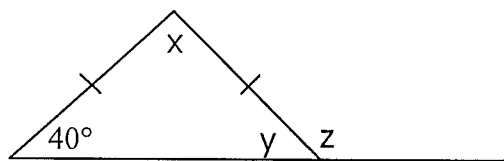- each $\angle$ is $90°$

Rhombus

- parallelogram w 4 equal sides
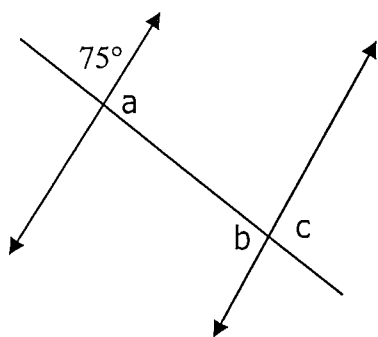
Square

- rhombus with 4 right $\angle$'s

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Practice. Find the missing angles and give the best reason.

$y = 40°$ ∡'s opposite = sides are =

$x = 106°$ ∡'s in a △ add up to 180°

$z = 140°$ Supplementary /∡'s on a line add up to 180°

$a = 105°$ supplementary /∡'s on a line add up to 180°

$b = 105°$ alt-interior ∡'s are =

$c = 105°$ vertically opposite ∡'s are =
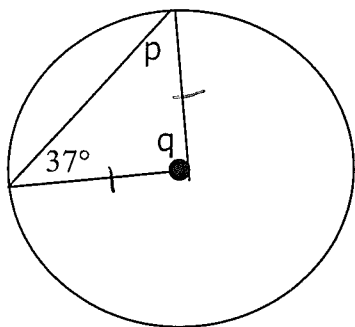
lines from centre to circle are = (radius)

∡$p = 37°$ ∡'s opposite = sides are =

∡$q = 106°$ ∡'s in a △ add up to 180°
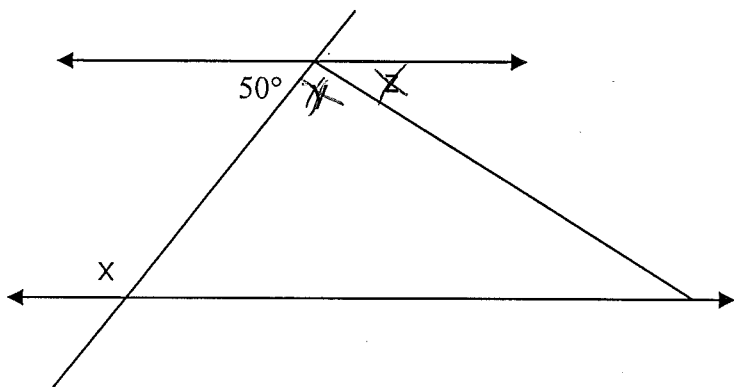
∡$x = 130°$ co-interior ∡'s add up to 180°

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Math 9   The Chord Property   Name:

Definitions:

Circle a set of points that are the same distance from the centre
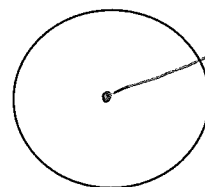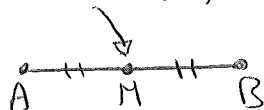
Radius distance from centre to circle

Perpendicular:

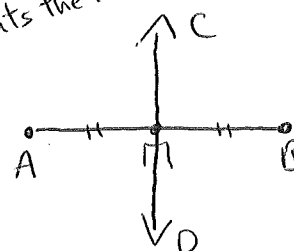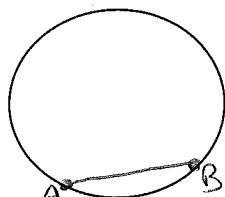intersecting lines @ right angles

Midpoint: (M)

A   M   B

Bisect: ~ cuts the line in half

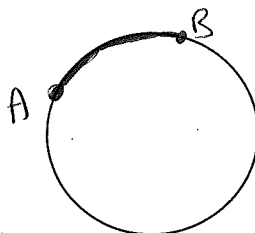A   M   B
C
D

CD bisects AB

Chord:

A   B

a line joining two Segment whose end points are on a circle

Arc:

A   B

part of a circle

$\overset{\frown}{AB}$

Draw a chord AB on this circle. Connect the midpoint M of the chord to the center O. Measure $\angle OMA$ and $\angle OMB$. Record your result

O

A   M   B
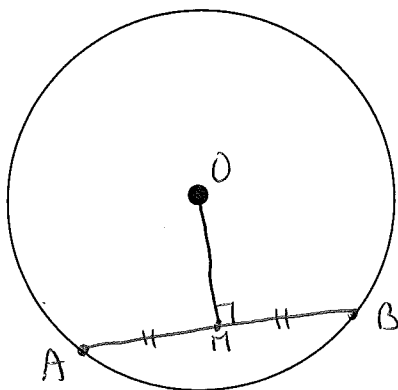
$\angle OMA = 90°$ $\angle OMB = 90°$

• a line from the centre bisects the chord AB creating perpendicular lines

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Draw a different length chord CD, in a different location. Draw a segment from the centre to the chord at right angles to the chord. Label the point where they intersect M. Measure CM and DM. What did you find?
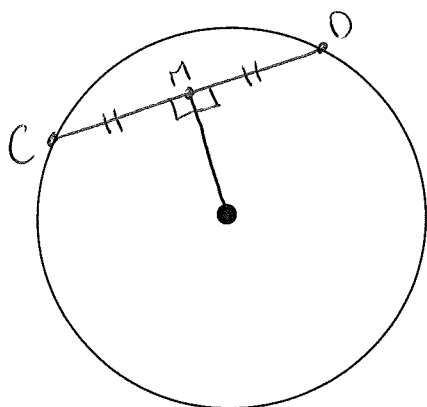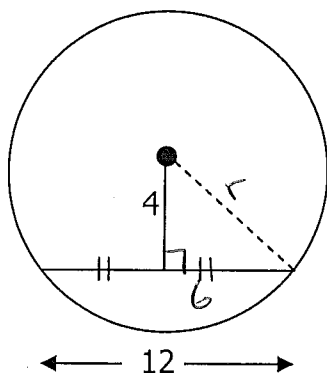

$CM = DM$

$\therefore OM$ is a perpendicular '$\perp$' bisector

Practice:

Find the length of the radius.
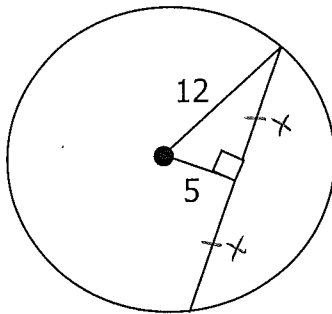

— use pythagoras $a^2 + b^2 = c^2$

$r^2 = 4^2 + 6^2$

$r^2 = 16 + 36$

$r^2 = 52$

$\sqrt{r^2} = \sqrt{52}$

$r \approx 7.21$

Find the length of the chord

$$x^2 + 5^2 = 12^2$$

$$x^2 + 25 = 144$$

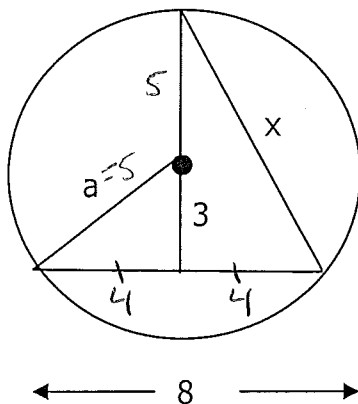$$ -25 \quad -25$$

$$x^2 = 119$$

$$\sqrt{x^2} = \sqrt{119}$$

$$x \approx 10.9$$

$$\therefore \text{chord} = 2 \times 10.9$$
$$= 21.8$$

Find the value of x

step 1

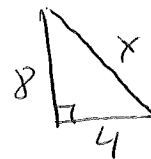$$a^2 = 3^2 + 4^2$$
$$a^2 = 9 + 16$$
$$a^2 = 25$$
$$\sqrt{a^2} = \sqrt{25}$$
$$a = 5$$
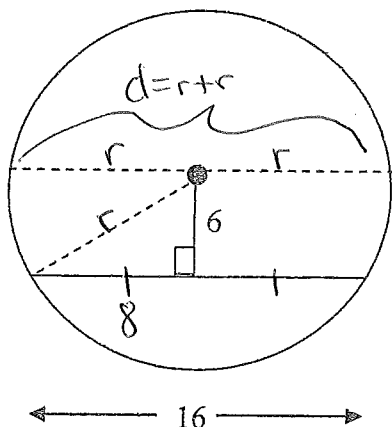$$\text{↳ radius}$$

step 2

$$x^2 = 4^2 + 8^2$$
$$x^2 = 16 + 64$$
$$x^2 = 80$$
$$\sqrt{x^2} = \sqrt{80}$$
$$x \approx 8.9$$

1)

Find the length of the diameter
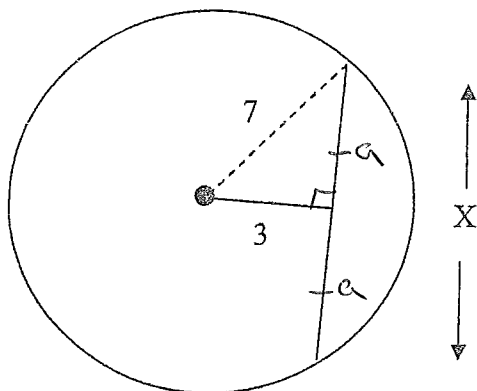HINT: Find the radius first

$$r^2 = 6^2 + 8^2$$
$$r^2 = 36 + 64$$
$$r^2 = 100$$
$$r = 10$$

$$d = r + r$$
$$= 10 + 10$$
$$= 20$$

2)

Find the length of the chord
HINT: Find half of the chord first

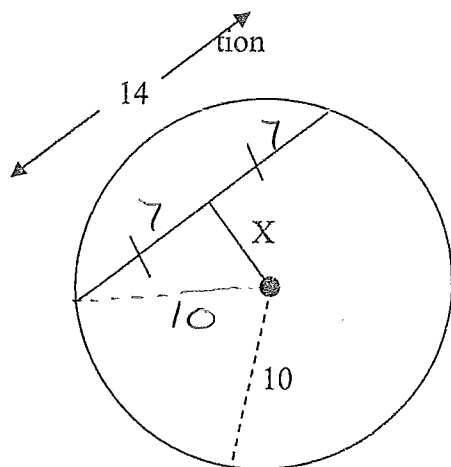$$7^2 = 3^2 + a^2$$
$$49 = 9 + a^2$$
$$ -9 \quad -9$$
$$40 = a^2$$
$$\sqrt{40} = \sqrt{a^2}$$
$$6.3 = a$$

$$x = a + a$$
$$= 6.3 + 6.3$$
$$= 12.6$$

3)

Find the length of X
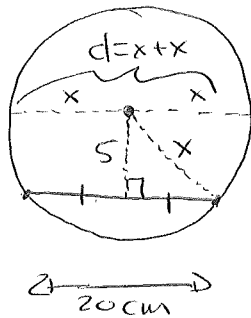HINT: Create a right triangle

$$10^2 = 7^2 + x^2$$
$$100 = 49 + x^2$$
$$ -49 \quad -49$$
$$\sqrt{51} = \sqrt{x^2}$$
$$7.1 = x$$

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4) A cylindrical pipe has a circular cross section. The water at the bottom of the pipe is 20 cm wide. The water is 5 cm from the center of the pipe. What is the diameter of the pipe?

$$x^2 = 5^2 + 10^2$$
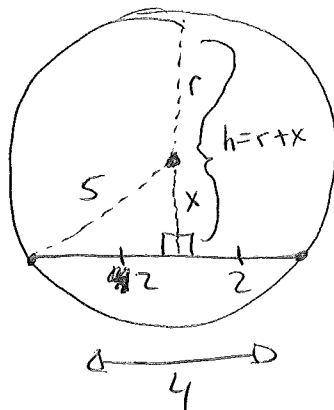$$x^2 = 25 + 100$$
$$\sqrt{x^2} = \sqrt{125}$$
$$x = 11.2 \text{ cm}$$

$$d = x + x$$
$$= 11.2 + 11.2$$
$$= 22.4 \text{ cm}$$

5)A road underpass is shaped as a circle. The underpass has a radius of 5 feet, while the pathway inside is 4 feet wide. What is the maximum height of the underpass?

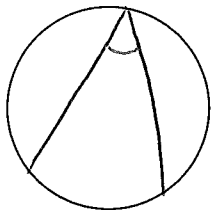

$$5^2 = x^2 + 2^2$$
$$25 = x^2 + 4$$
$$ -4 -4$$
$$\sqrt{21} = \sqrt{x^2}$$
$$4.6 = x$$

$$\text{height} = x + \text{radius}$$
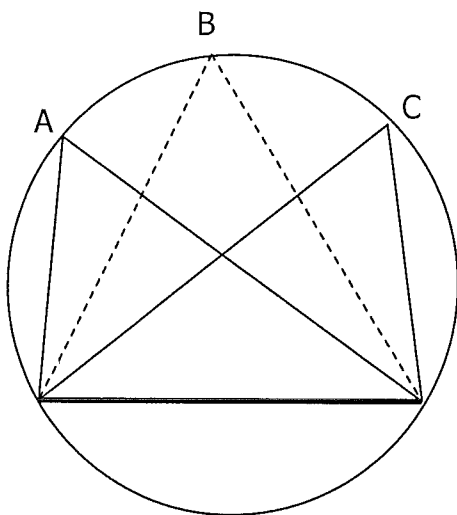$$= 4.6 + 5$$
$$= 9.6 \text{ ft.}$$

Math 9   Inscribed Angles   Name:

Definitions: Inscribed angle: is an angle whose "vertex" is on the circle

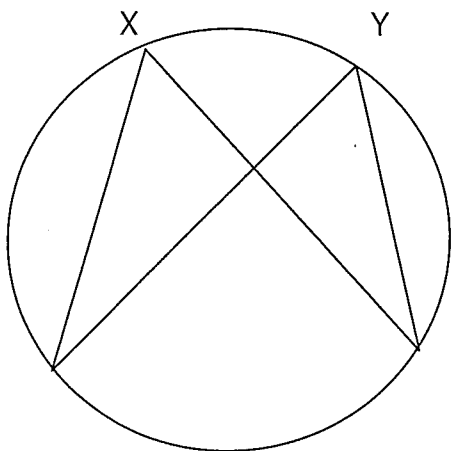Three hockey players face the net in a circular rink. Which player has the best angle at which to shoot?

$\angle A = 60°$

$\angle B = 60°$

$\angle C = 60°$

∴ all 3 inscribed ∠'s are =

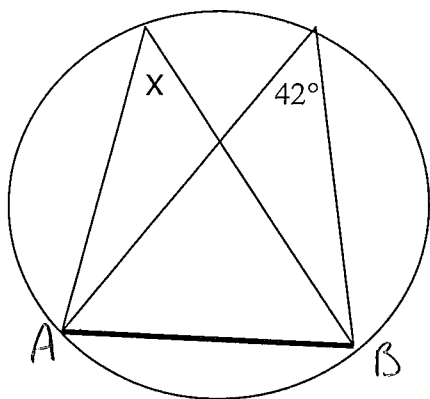Measure the inscribed angles X and Y. What can you conclude?

$\angle X = 59°$

$\angle Y = 59°$
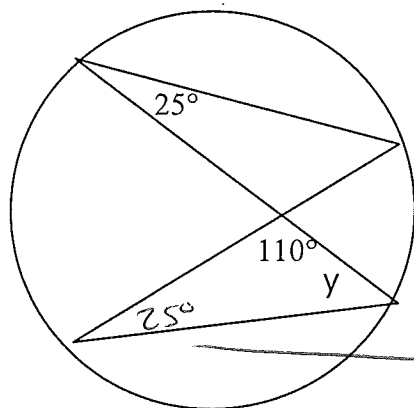
∴ inscribed ∠'s covered by the same chord or arc are equal

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Practice:

$\angle x = 42°$
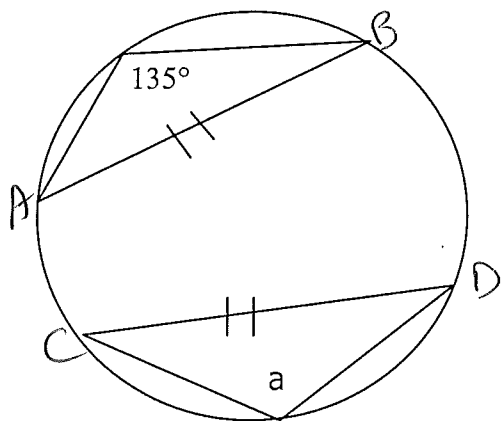
↳ both are inscribed angles 'sharing' the same chord AB

→ shares an arc with 25°

∴ $\angle y = 45°$

$(180° - 110° - 25° = 45°$

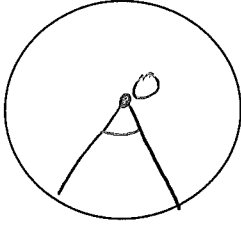∠'s in △ add to 180°)

$\angle a = 135°$

★ both inscribed angles are 'covered by' equal chords $\overline{AB}$ and $\overline{CD}$

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Definition: Central Angle —▷ an angle whose vertex is
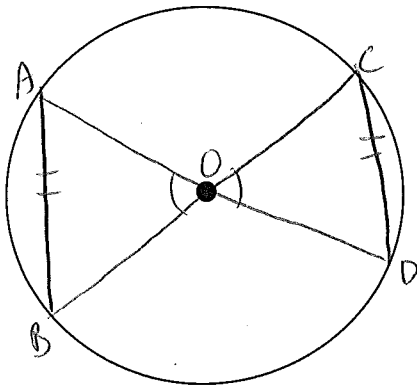   at the centre

Draw two equal chords, one on either side of the circle. Label them AB and CD.
Label center as O. Create central angles $\angle AOB$ and $\angle COD$ by joining A and B to O.
and C and D to O.

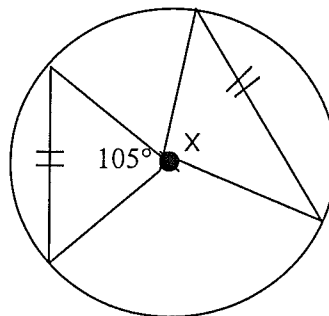Measure $\angle AOB$ and $\angle COD$. What is your conclusion?

$\angle AOB = 60°$ $\angle COD = 60°$

Conclusion: ∴ both $\angle AOB$ & $\angle COD$ are equal
                      ↳ central ∠'s with equal
                            chords will have = ∠'s
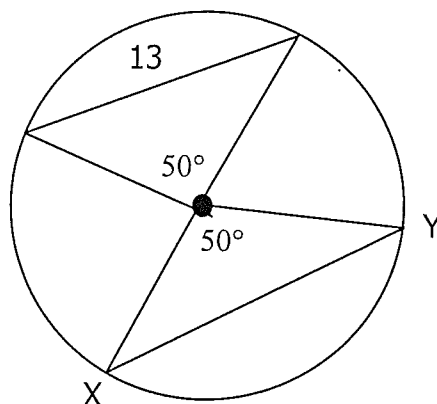

Practice:

1. Find the measure of x

$\angle x = 105°$

both central angles
are 'covered by' = chords

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Find the length of chord XY

$XY = 13$
  ∟central ∠'s are =,
   thus chords must
   be = length
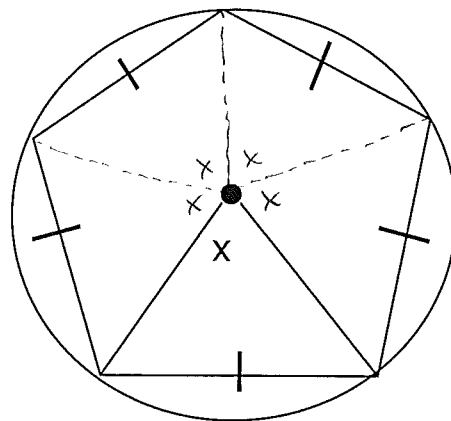

Find the measure of X

angles around a pt add to 360°

$X + X + X + X + X = 360°$

$$\frac{5x}{5} = \frac{360°}{5}$$

$X = 72°$

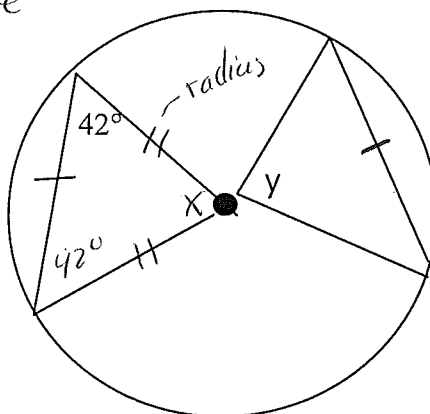Find the measure of Y   ∠'s are = opposite = sides

$∢X = 180° - 42° - 42°$
   $= 96°$

∴ $∢Y$ is also $96°$
   ∟ equal chords

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Math 9   Central/Inscribed Property   Name:

This lesson is designed to discover the relationship between and central and an inscribed angle

Draw chord AC. Draw inscribed angle ABC. Draw central angle AOC.
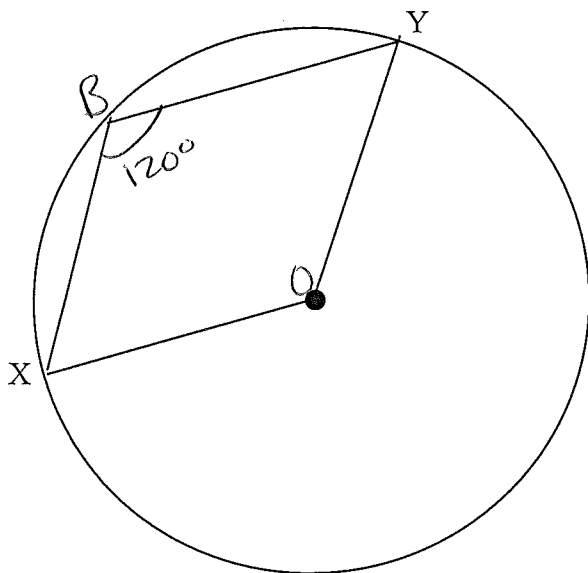Measure $\angle ABC$ and $\angle AOC$. Record your results.
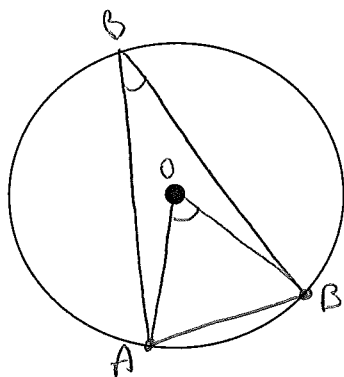
Both ∠'s "share" the same chord.

$\angle ABC =$ __37°__ $\angle AOC =$ __74°__
           inscribed ∠   central ∠

Conclusion:

The central ∠, when ~~connected~~ sharing the same chord, was double the size of the inscribed ∠

Repeat this experiment using an arc XY.

∠XBY (inscribed) = 120°

∠XOY (central) = 2 × 120°
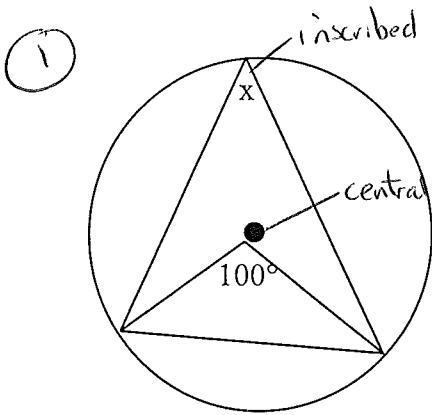   = 240°

∴ the same conclusion as before works on equal/same arcs

Do you have the same result? yes
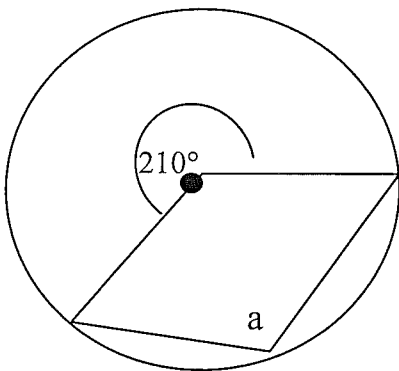
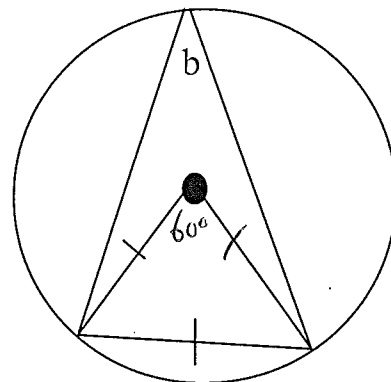central ∠ is 2× inscribed ∠ on equal/same chords/arcs.

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Practice:

①

inscribed

x

central

100°

∠x = 100° ÷ 2
= 50° (inscribed ½ of central ∠)

②

47°

y

z

47°

∠'s opp = sides are =

∠y = 180-47-47
= 86° (∠'s in a △)

∠z = 86 ÷ 2
= 43°

210°

a

∠a = 210° ÷ 2
= 105°

b

60°
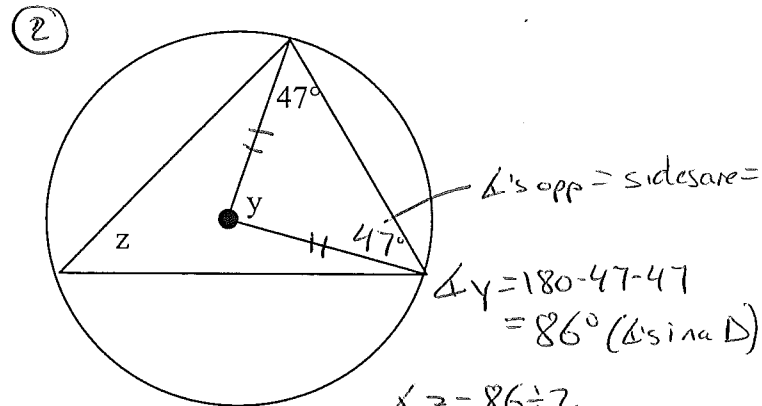
→ equilateral △, all ∠'s 60°

→ ∠b = 60° ÷ 2 = 30°

41°

p

n
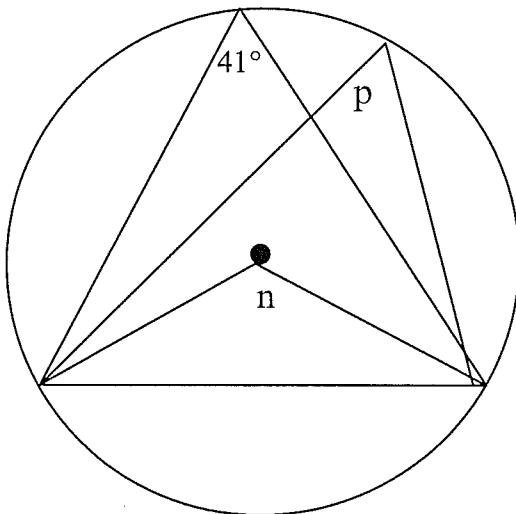
∠p = 41° (inscribed ∠ on same chord)

∠n = 2 × 41°
= 82° (central ∠ double inscribed ∠)

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Math 9   Semi-Circle Property   Name:

In this lesson we will explore the properties of an inscribed angle covered by the diameter

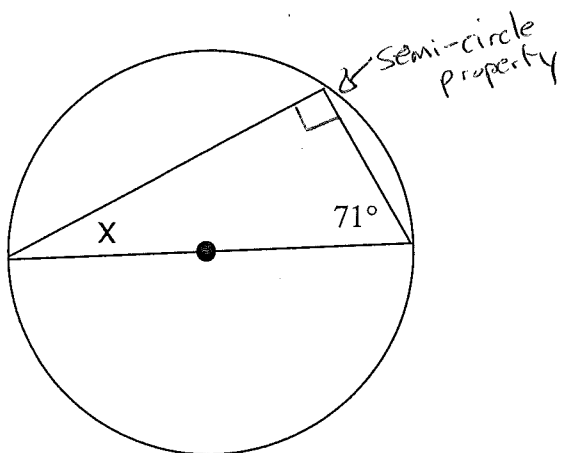Draw inscribed angle APB. Measure and record $\angle APB$ $\angle APB =$ __90°__
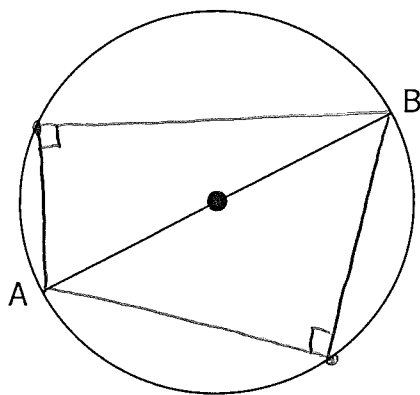
Draw inscribed angle AQB. Measure and record $\angle AQB$ $\angle AQB =$ __90°__

Conclusion: an inscribed $\angle$ in a semi-circle is 90°

Practice:

Semi-circle property

71°

X

$\triangle$ Sum = 180°

$\angle x = 180° - 90° - 71°$

$= 19°$

semi-circle property

Y

$y + y + 90° = 180°$

$2y + 90° = 180°$

$2y = 90°$

$\dfrac{2y}{2} = \dfrac{90°}{2}$

$y = 45°$

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$\triangle$ sum = 180°

$$3x + 2x + 90° = 180°$$
$$5x + 90° = 180°$$
$$\frac{5x}{5} = \frac{90°}{5}$$
$$x = 18°$$
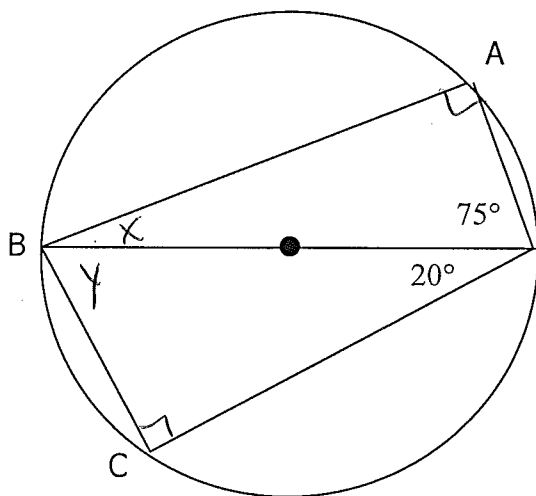
$\angle A = 3x$
$\quad = 3(18°)$
$\quad = 54°$

X = _____ 18° _____

$\angle x = 180° - 90° - 75°$
$\quad = 15°$

$\angle y = 180° - 90° - 20°$
$\quad = 70°$

$\angle ABC = \underline{\angle x + \angle y}$
$\quad = 15° + 70°$
$\quad = 85°$
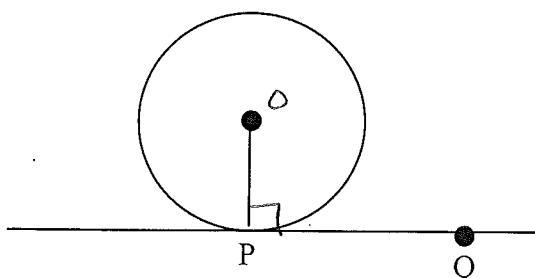
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Ma 9   Tangent Radius Property   Name:

Goal is to explore the relationship between a tangent and its radius

Definition: A tangent is a line that intersects a circle @ a single point 'p'
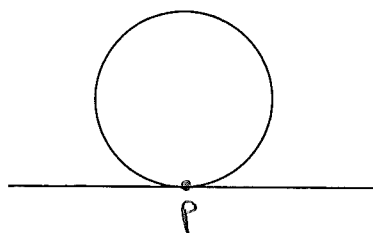
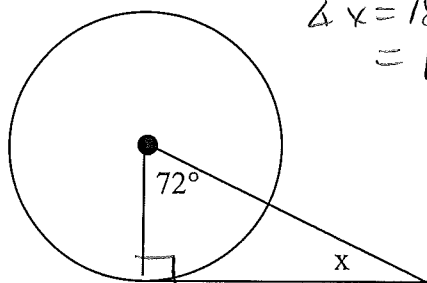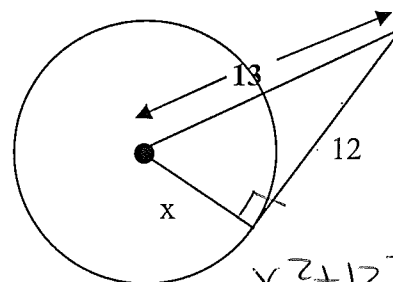Draw a segment joining the center with the point of tangency. Measure angle OPQ

$\angle OPQ = \underline{90^\circ}$

Conclusion:

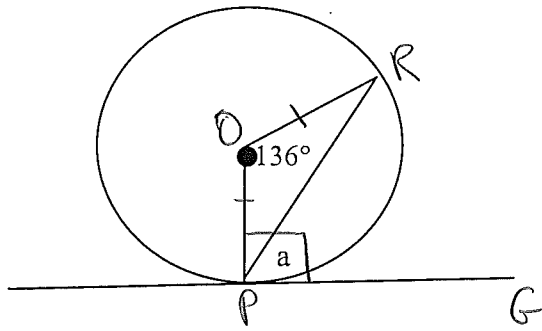The angle between the <u>tangent</u> and the <u>radius</u> is 90°

Practice:

$\triangle$ sum $= 180^\circ$

$\angle x = 180^\circ - 90^\circ - 72^\circ$

$= 18^\circ$

$x^2 + 12^2 = 13^2$

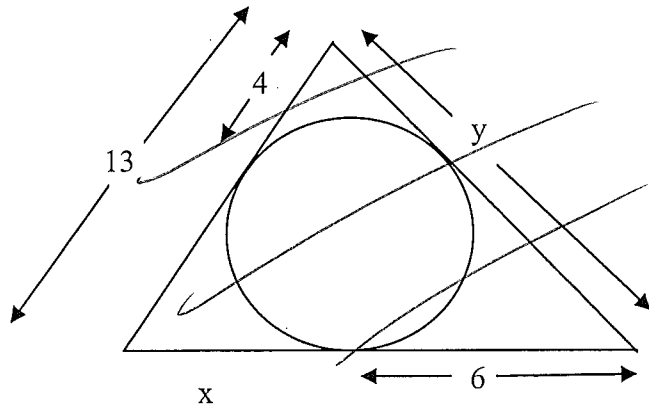$x^2 + 144 = 169$

$x^2 = 25$

$x = 5$

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Step 1: $\angle OPG = 90°$

Step 2: $\triangle POR$ is isosceles (2 equal sides) ← radius

$\therefore \angle P \text{ \& } \angle R = \left(\dfrac{180° - 136°}{2}\right)$
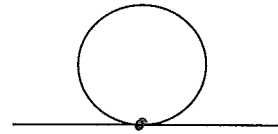
$= 22°$

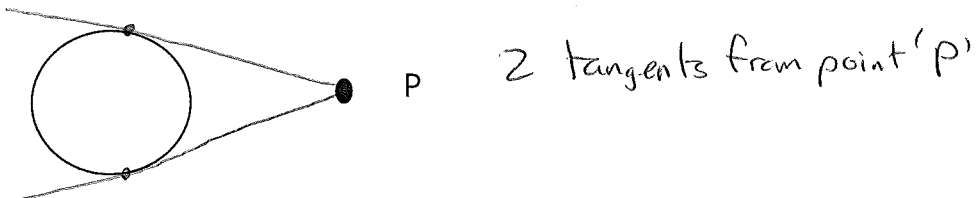Step 3: $\angle a = 90° - 22°$

$= 68°$

Perimeter =

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The goal is to explore the relationship between tangents drawn to a circle

Remember---A tangent is a line that intersects a circle once

How many tangents can be drawn to a circle from point P?

P   2 tangents from point 'P'

Draw tangents from point P to the circle. Label the points of tangency as A and B. Measure the lengths of PA and PB. Record the result. What is your conclusion?
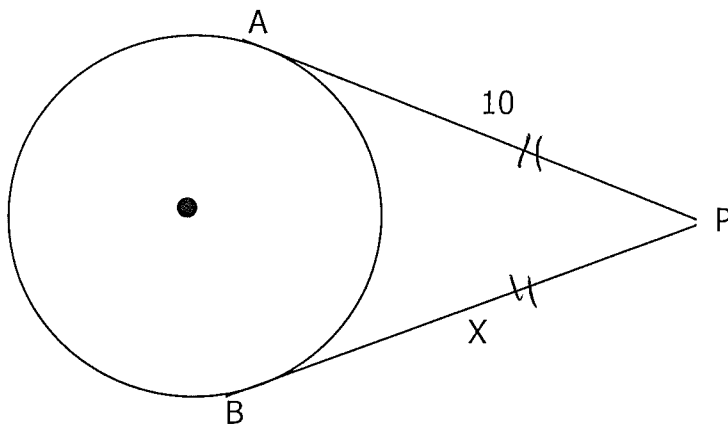
A

H

P

PA = ___4___

PB = ___4___

B
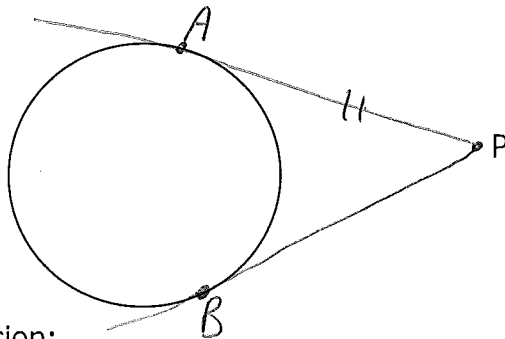
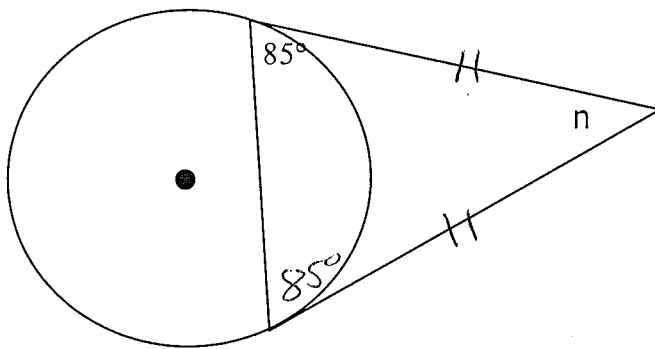Conclusion:

The distance from P to A or B are equal

Equal Tangents Property:

Tangents drawn to a circle, from an external point, are equal in length
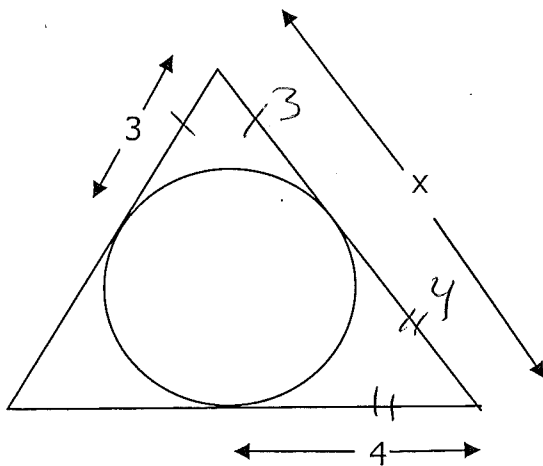
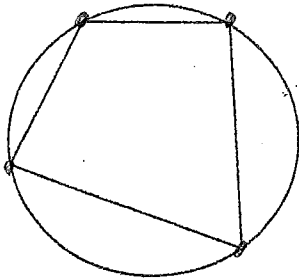Practice:

$X = 10$

isosceles triangle → equal tangents

$\angle n = 180° - 85° - 85°$

$= 10°$

$X = 3 + 4$

$X = 7$

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The goal is to explore the angles in a Cyclic Quadrilateral

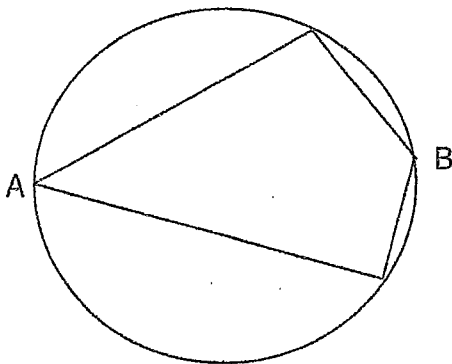Definition: A Cyclic Quadrilateral is: a 4-sided figure whose vertices are on the circle.
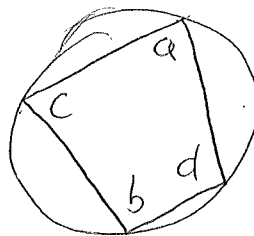
Measure angles A and B. Record your results
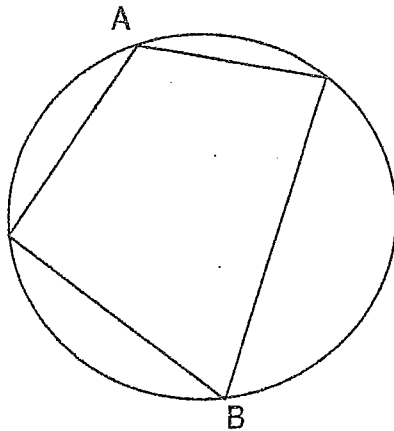
$\angle A = 115°$
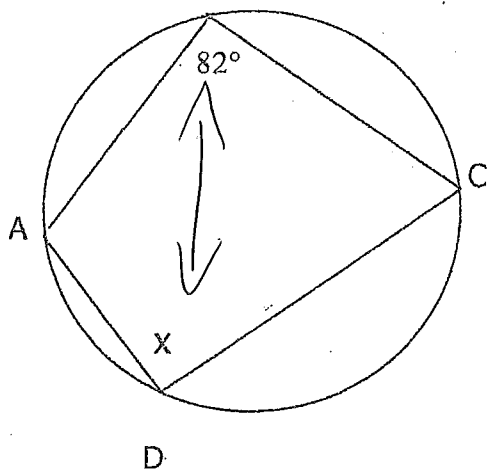
$\angle B = 65°$

$\angle A = 45°$ $\angle B = 135°$

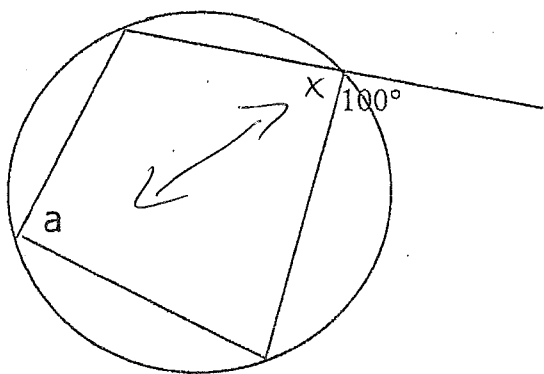Cyclic Quadrilateral Property: Opposite inscribed angles add to $180°$

$\angle a + b = 180°$

$\angle c + d = 180°$

Practice:

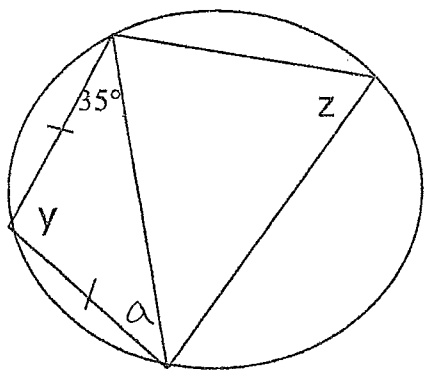$\angle x = 180° - 82°$
$= 98°$

① $\angle x = 80°$ ($\angle's$ on a line)

② $\angle a = 180° - 80°$
$= 100°$

① $\angle a = 35°$ (Isosceles $\triangle$)

② $\angle y = 180° - 35° - 35°$ ($\angle's$ in a $\triangle$)
$= 110°$

③ $\angle z = 180° - 110°$
$= 70°$
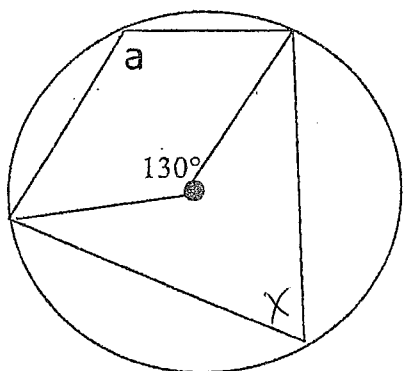
① $\angle x = 65°$ (inscribed $\angle$ is $\frac{1}{2}$ central $\angle$)

② $\angle a = 180° - 65°$
$= 115°$

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