2.8 Estimating Square Roots

YOU WILL NEED

- a calculator

GOAL

Use perfect square benchmarks to estimate square roots of other fractions and decimals.

INVESTIGATE the Math

Bay is preparing for the Egg Drop Experiment in science class. Bay will try to drop the egg 23.7 m, without breaking it. He needs to determine how long an egg will take to hit the ground. He will estimate the drop time for the egg using the formula $time = 0.45\sqrt{height}$, where time is measured in seconds and height in metres.

? How long will it take an egg to hit the ground?

A. Substitute the known value into the formula.

B. What is the greatest perfect square less than the height? What is the least perfect square greater than the height?

C. Which of the two numbers you found in part B is the given height closer to?

D. Estimate the square root of the height to one decimal place using the numbers from part B as benchmarks. Check your answer by multiplying and estimate again if you need to.

E. Determine $\sqrt{23.7}$ m to two decimal places using a calculator.

F. Write 23.7 as an improper fraction. Is the square root of 23.7 an exact value? Explain how you know.

G. How long will this egg take to hit the ground, to one decimal place?

Reflecting

H. Why is it helpful to estimate the square root of a number that is not a perfect square?

WORK WITH the Math

EXAMPLE 1 | Estimating a square root to verify a calculation

Shelby knew that square root problems involve two identical numbers, so she said $\sqrt{110} = 55$. Is her answer reasonable?

Shelby's Solution

$\sqrt{110} = 55$ — I decided to estimate. I know 110 isn't a perfect square, so I thought of square numbers that are less than 110 and greater than 110.

100 ------------ 110 ------------ 121

$\sqrt{100} = 10 \qquad \sqrt{121} = 11$ — I compared the square roots of these numbers to my estimate.

$\sqrt{110}$ is between 10 and 11, so my estimate of $\sqrt{110} = 55$ is not reasonable.

Yvonne's Solution

$\sqrt{110} = 55?$ — I squared the answer on my calculator.

$55^2 = 3025$

Obviously, $3025 \neq 110$, so the square root of 110 is not 55. The answer is not reasonable.

EXAMPLE 2 | Estimating a square root by reasoning

Estimate $\sqrt{0.84}$.

Nicole's Solution

0.84 is $\frac{84}{100}$ — I thought of the decimal in hundredths and looked for a square root that I knew that was close to it.

That is close to $\frac{81}{100}$, and its square root is $\frac{9}{10}$, or 0.9.

I estimated 0.92 as the square root. — Since $0.84 > 0.81$, I chose a number a little greater than 0.9.

$0.92 \times 0.92 = 0.8464$ — I checked my estimate by squaring it.

$\sqrt{0.84}$ is about 0.92 — My estimate is not an exact value.

EXAMPLE 3 | Reasoning about square roots of decimals

Is either of these square roots an exact value: $\sqrt{0.49}$, $\sqrt{4.9}$? Evaluate each.

Bay's Solution

$$\sqrt{0.49} = \sqrt{\frac{49}{100}} = \frac{\sqrt{49}}{\sqrt{100}} = \frac{7}{10} \text{ or } 0.7$$

I can write 0.49 as a fraction where the numerator and denominator are perfect squares, so $\sqrt{0.49}$ is an exact value.

$$0.7 \times 0.7 = 0.49$$

I checked by multiplying.

$$\sqrt{4.9} = \sqrt{\frac{49}{10}} = \frac{\sqrt{49}}{\sqrt{10}}$$

or

$$\sqrt{4.9} = \sqrt{\frac{490}{100}} = \frac{\sqrt{490}}{\sqrt{100}}$$

I cannot write 4.9 as a fraction where the numerator and denominator are perfect squares.
In my first try, the numerator is a perfect square, but the denominator is not.
In my second try, the denominator is a perfect square, but the numerator is not, so $\sqrt{4.9}$ is not an exact value.

$4 < 4.9 < 9$ so the square root of 4.9 is a decimal between 2 and 3.

I chose perfect square benchmarks of 4 and 9 to estimate $\sqrt{4.9}$.

$$\sqrt{4.9} \doteq 2.2$$

Because 4.9 is much closer to 4 than to 9, I estimated a decimal value close to 2.

$$2.2 \times 2.2 = 4.84$$

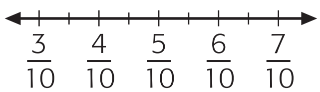
[√] [4] [.] [9] [=] 2.2135943

I checked by multiplying.
Then I compared my estimate to the value determined using a calculator.
My estimate was reasonable.

EXAMPLE 4 | Identifying a square root between two numbers

The area of a square is between $\frac{3}{10}$ units^2 and $\frac{7}{10}$ units^2. What might the side length of the square be?

Derek's Solution: Using a number line



I needed a value between $\frac{3}{10}$ and $\frac{7}{10}$.
I looked for a number whose square root would be easy to calculate.

$\frac{3}{10} = \frac{30}{100}$ and $\frac{7}{10} = \frac{70}{100}$, so $\frac{36}{100}$ is between $\frac{3}{10}$ and $\frac{7}{10}$.

$\sqrt{\frac{36}{100}} = \frac{6}{10}$ — I took the square root.

The side length of the square might be $\frac{6}{10}$ units.

Austin's Solution: Using reasoning

$\frac{3}{10} = 3 \div 10 = 0.3$

$\frac{7}{10} = 7 \div 10 = 0.7$

I wrote $\frac{3}{10}$ and $\frac{7}{10}$ as decimals.

$0.3 < 0.5 < 0.7$

[√] [0] [.] [5] [=] [.7071067]

I chose a number between 0.3 and 0.7 and determined the square root with my calculator. Then I used a nearby estimate.

The side length of the square might be $\frac{7}{10}$ units.

In Summary

Key Idea

- You can use perfect squares as benchmarks to estimate the square root of numbers that are not perfect squares. For example, to estimate $\sqrt{259}$, think that 16^2 is 256 and 17^2 is 289, so $\sqrt{259}$ must be closer to 16 than 17, or about 16.1.

Need to Know

- You can check the square root of a number by multiplying the square root by itself, or squaring it.
- Decimals that cannot be written as equivalent fractions with numerators and denominators that are both perfect squares have square roots that are not exact values.

Checking

1. List the two closest whole numbers between which each square root lies.

a) $\sqrt{8.5}$ **b)** $\sqrt{52.4}$ **c)** $\sqrt{149.7}$ **d)** $\sqrt{\frac{5}{9}}$

2. Estimate each square root in question 1 to two decimal places using your calculator.

3. How do you know your answers to question 2 are reasonable?

Practising

4. Calculate the side length of a square with an area of $6.4\ cm^2$.

5. A square has an area of 31.5 cm^2. Estimate the side length of the square. Explain how you estimated.

6. The areas of some squares are shown. Estimate the length of the sides of each square. Then, determine the lengths using a calculator.

   a) 1.44 $units^2$ **c)** 0.01 $units^2$ **e)** $\frac{16}{144}$ $units^2$

   b) 75.6 $units^2$ **d)** $\frac{1}{4}$ $units^2$ **f)** $\frac{36}{25}$ $units^2$

7. **Multiple choice.** Between which two whole numbers does $\sqrt{26.7}$ lie?

   A. 25 and 30 **B.** 10 and 20 **C.** 5 and 6 **D.** none of these

8. **Multiple choice.** Calculate the side length of a square with an area of 6.4 cm^2.

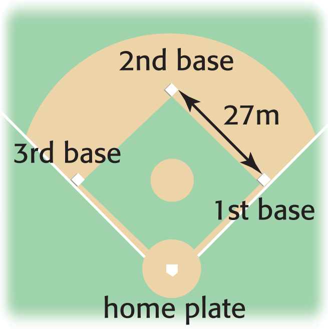
   A. 1.6 cm **B.** 40.96 cm **C.** 2.5 cm **D.** 0.8 cm

9. Pearl is going to paint her bedroom wall pink. The wall is 2.5 m by 2.5 m. She has bought a can of paint that will cover 20 m^2.
   a) Estimate to determine if she has enough paint for two coats. Show your work.
   b) What is the side length of the largest square she can paint with two coats? Answer to the nearest metre.

10. A square-based shed has a floor area of 50.6 m^2. Which estimate is closer to the length of the front of the shed: 7.2 m or 7.7 m? Explain how you can answer this without using a calculator.

11. **a)** How do you know that $\sqrt{0.7} > 0.8$?
    b) Will the square root of a decimal always be greater than the square root of the decimal that is 0.1 greater? Explain.

12. Explain how you know that $\sqrt{6.4}$ cannot be 0.8 or 0.08.



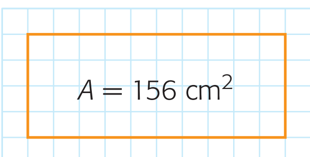
13. A baseball diamond is a square with a side length of about 27 m. Joe throws the ball from second base to home plate. Estimate how far Joe threw the ball.

Closing

14. It's sometimes easier to calculate the square root of a decimal hundredth than a decimal tenth without a calculator, for example, 1.44 than 14.4. Is the same true for estimating?

Extending

15. Hedy estimated $\sqrt{2358}$ as 50. Explain how you could give a closer estimate.



16. The area of the rectangle is 156 cm^2. Divide the rectangle into squares to determine the approximate length of each side. Describe why you chose the strategy you used.