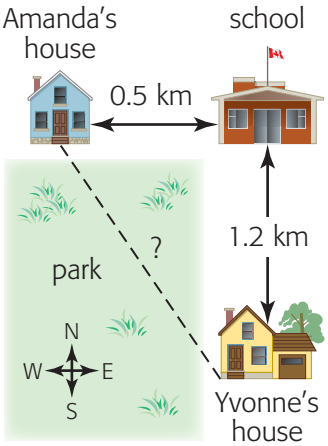
2.7 Calculating Square Roots

YOU WILL NEED

- a calculator



GOAL

Calculate the square roots of fractions and decimals.

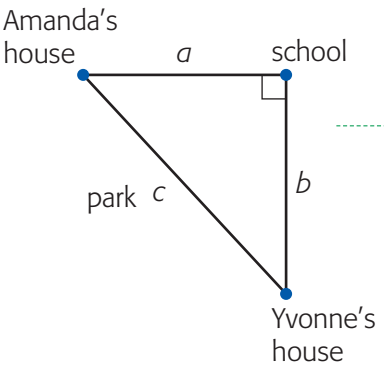
LEARN ABOUT the Math

Amanda walks 0.5 km east to go to school. On Thursdays, she goes to Yvonne's house after school to play video games. Yvonne's house is 1.2 km south of the school. Amanda cuts through the park to get home.

How far does Amanda walk to get home from Yvonne's house?

EXAMPLE 1 | Applying the Pythagorean theorem

Amanda's Solution



The triangle is a right triangle. The distance across the park is the hypotenuse, so it must be greater than 1.2 km.

$c^2 = a^2 + b^2$
$c^2 = (0.5)^2 + (1.2)^2$
$c^2 = 0.25 + 1.44$
$c^2 = 1.69$

I used the Pythagorean theorem to write a relationship between the sides in the right triangle. I solved for c^2.

$c = \sqrt{1.69}$

√ 1 . 6 9 = 1.3

I calculated the square root on my calculator.

$c = 1.3$ km

1 . 3 ^ 2 = 1.69

I checked my answer by squaring. The square root is an exact value since squaring it results in the number I started with.

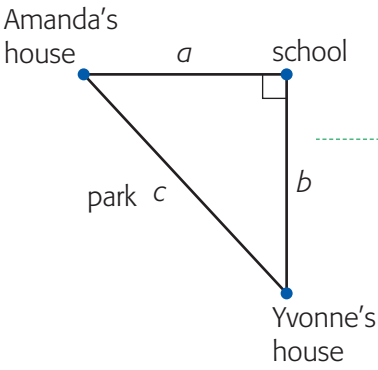
It's 1.3 km between our houses.

My answer seems reasonable because the distance across the park is greater than 1.2 km.

Technology *Tip*

Different calculators calculate square roots in different ways. With some, you press √ first and then enter the number. With others, you enter the number first and then press √. There are other ways too.

Yvonne's Solution: Using Fractions



The triangle is a right triangle. The distance across the park is the hypotenuse, so it must be the longest side of the triangle.

$c^2 = a^2 + b^2$

$c^2 = \left(\frac{5}{10}\right)^2 + \left(\frac{12}{10}\right)^2$

$c^2 = \frac{25}{100} + \frac{144}{100}$

I used the Pythagorean theorem to write a relationship between the sides in the right triangle. I wrote the decimals as fractions, where $0.5 = \frac{5}{10}$ and $1.2 = \frac{12}{10}$. Then I solved for c^2.

$c^2 = \frac{169}{100}$

$c = \sqrt{\frac{169}{100}}$

$c = \frac{\sqrt{169}}{\sqrt{100}}$

I took the square root of both sides. The square root must be a fraction, $\frac{a}{b}$, where $\frac{169}{100} = \frac{a \times a}{b \times b}$. Since this is equivalent to $\frac{169}{100} = \frac{a^2}{b^2}$, I reasoned that $\frac{\sqrt{169}}{\sqrt{100}} = \frac{a}{b}$.

$c = \frac{13}{10}$

$c = 1\frac{3}{10}$ or 1.3 km

Then I evaluated the square roots of the numerator and the denominator.

$\frac{13}{10} \times \frac{13}{10} = \frac{169}{100}$

I checked my answer by multiplying. My square root is an exact value.

It's 1.3 km between our houses.

My answer seems reasonable because this distance is greater than 1.2 km

Reflecting

A. How are Amanda's and Yvonne's methods similar? How are they different?

B. Do you prefer Amanda's method or Yvonne's method? Explain why.

C. Explain how Yvonne determined the square root of the fraction in her solution.

WORK WITH the Math

EXAMPLE 2 | **Determining the square root of decimals greater than and less than 1**

When is the square root of a number greater than the number?

Bay's Solution

I know that the side length of a square with an area of 1 square unit is 1 unit. I'll try squares with greater and lesser areas.

$\sqrt{1} = 1$

I calculated the side length of a square with an area greater than 1. The square root is less than the number.

$\sqrt{1.21} = 1.1$

I calculated the side length of a square with an area of less than 1 square unit. The square root is greater than the number.

$\sqrt{0.64} = 0.8$

The square root of a number is greater than the number when the number is between 0 and 1.

I think this is going to happen for all squares whose sides have length greater than 0 but less than 1.

EXAMPLE 3 | **Determining the square root of a fraction using a quotient**

Austin is building a patio using square concrete patio slabs; 25 of the slabs cover 9 m^2. What are the dimensions of the top of each slab?

Austin's Solution

The top of each slab has an area of $\frac{9}{25}$. — I divided to determine the area of one top.

The length of each side is $\sqrt{\frac{9}{25}}$. — Since the top is a square, I know the length and width must be equal. I can calculate the length of each side by determining the square root of its area.

$\sqrt{\frac{9}{25}} = \frac{\sqrt{9}}{\sqrt{25}}$ — The square root of a quotient is the same as the quotient of the square roots.

$= \frac{3}{5}$ — I calculated the square root of the numerator and the denominator.

$= 0.6$ — I wrote the fraction as a decimal.

The top of the slab has dimensions of 0.6 m by 0.6 m.

EXAMPLE 4 Using order of operations with a square root

Calculate $2^4 \times \sqrt{36} + 4^2 \div 2 + 1$.

Amanda's Solution

$2^4 \times \sqrt{36} + 4^2 \div 2 + 1$
$= 16 \times 6 + 16 \div 2 + 1$
$= 96 + 8 + 1$
$= 105$

I treated the square root like a power.
I evaluated the powers first.
Then, I divided and multiplied.
Lastly, I added.

In Summary

Key Idea

- If a positive number is less than 1, then its square root will be greater than the original number. If a positive number is greater than 1, then its square root will be less than the original number.

Need to Know

- The square root of a quotient equals the quotient of the square roots. $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
- If the numerator and denominator of a fraction are both perfect squares, then the square root of the fraction is an exact value.
- If a decimal can be written as an equivalent fraction whose numerator and denominator are perfect squares, then the square root of the decimal is an exact value.

Checking

1. Enter the missing numbers.

a) $\sqrt{49} = \sqrt{\square \times \square}$ **c)** $\sqrt{\frac{4}{9}} = \frac{\square}{3}$ **e)** $\frac{\sqrt{144}}{\sqrt{225}} = \frac{\square}{\square}$

b) $\sqrt{\square} = 11$ **d)** $\sqrt{\frac{\square}{81}} = \frac{7}{9}$ **f)** $\frac{\sqrt{\square}}{\sqrt{\square}} = \frac{10}{13}$

2. A square field has an area of 1.44 km^2. Calculate its length and width without a calculator. Show your work.

Practising

3. Enter the missing numbers.

a) $\sqrt{3.61} = \square$ **b)** $\sqrt{\square} = 0.07$ **c)** $\sqrt{\frac{100}{289}} = \square$ **d)** $\frac{\sqrt{\square}}{\sqrt{\square}} = \frac{4}{6}$

4. Calculate.

a) $\sqrt{\frac{9}{1}}$ **b)** $\sqrt{\frac{81}{9}}$ **c)** $\sqrt{\frac{729}{81}}$

5. Based on your answers to question 4, how can you predict the answer to $\sqrt{\frac{64}{16}}$?

6. Evaluate.

a) $7^2 + \sqrt{4} \times 4^2 - 2$ **b)** $(\sqrt{81} + \sqrt{64})^2 \div 17 + 6$

7. Multiple choice. Evaluate $\sqrt{\frac{121}{256}}$.

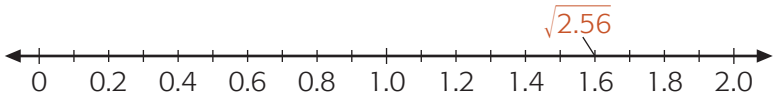
A. $\frac{11}{16}$ **B.** $\frac{121}{256}$ **C.** $\frac{14\,641}{65\,536}$ **D.** $\frac{16}{11}$

8. Determine each square root to one decimal place.

a) $\sqrt{2.56}$ **c)** $\sqrt{1.69}$ **e)** $\sqrt{0.8100}$ **g)** $\sqrt{0.36}$

b) $\sqrt{1.96}$ **d)** $\sqrt{1.44}$ **f)** $\sqrt{0.4900}$ **h)** $\sqrt{0.25}$

9. Label the square and square root from each part of question 10 on a number line. The first one is done for you.

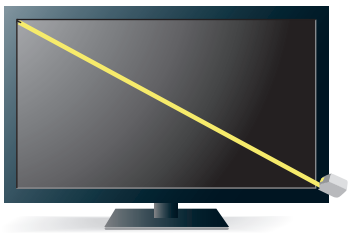


10. The square root of 1 is 1. That is, $\sqrt{1} = \sqrt{1 \times 1} = 1$. Is any other positive rational number equal to its square root? How do you know?

11. The square root of a number is 16.5. What is the number?

12. You know that $\sqrt{576} = 24$. What decimal square roots could you calculate easily using that information? Explain.

13. Bittu has a new TV with an 84 cm screen. He wants to put it above his fireplace in a space 150 cm wide and 75 cm high. Will the TV fit into the space?



14. **a)** Complete each statement.

A. $\sqrt{\frac{1}{4}} = \sqrt{0.\square} = \frac{1}{2}$ **B.** $\sqrt{0.04} = \frac{1}{\square}$ **C.** $\sqrt{0.\square} = \frac{1}{\square}$

b) Explain how you know your answers are reasonable.

15. Verify each statement and correct those that are incorrect.

a) $\sqrt{6.4} = 3.2$ **c)** $\sqrt{256} = 16$

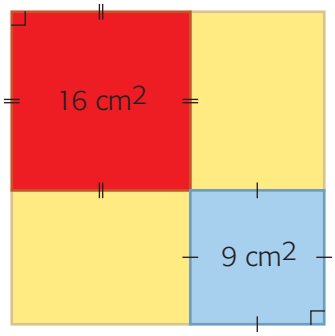
b) $\sqrt{0.9} = 0.03$ **d)** $\sqrt{0.25} = 0.5$

16. Calculate.

a) $\sqrt{0.09}$ **b)** $\sqrt{0.0009}$ **c)** $\sqrt{0.000\,009}$ **d)** $\sqrt{0.000\,000\,09}$

17. What pattern do you notice in the answers in the previous question?

18. What is the area of the yellow region?



Closing

19. Which of these have a square root that is an exact value: 0.49, 4.9, 0.0049? Explain.

Extending

20. Could $\sqrt{\frac{\square}{9}}$ be a fraction with a denominator of 2? Explain.

21. The area of a circle with radius r is πr^2, where $\pi \doteq 3.14$. What is the radius of a circle of area 50.24 cm^2?

22. The area of the square is 12.25 cm^2. Estimate the radius of the circle.