

2.6

Communicate about Calculations with Powers

YOU WILL NEED

- a calculator

WIN A TRIP FOR 2 TO BANFF Entry Form Name: _____ e-mail: _____ Phone no: _____ Answer the following skill-testing question: $4 + 5^3 - 3^2 + 8 \times (27 \div 9)^2$
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Communication Checklist

- ✓ Did you include all the steps?
- ✓ Did you explain why you did each step?
- ✓ Did you explain how you did each step?
- ✓ Did you justify your conclusion?

GOAL

Clearly explain the steps for calculating with powers.

INVESTIGATE the Math

Bay and Austin were answering this skill-testing question. Bay's answer was 1152 and Austin's answer was 192. Austin started to show Bay why his answer was correct, but then his cell phone rang and he was distracted. Here is his explanation.

Use order of operations.

$$\begin{aligned} 4 + 5^3 - 3^2 + 8 \times (27 \div 9)^2 & \quad \text{brackets first} \\ = 4 + 5^3 - 3^2 + 8 \times (3)^2 & \quad \text{then exponents} \\ = 4 + 125 - 9 + 8 \times 9 & \quad \text{divide/multiply} \\ = 4 + & \end{aligned}$$

? Why is this question a good test of mathematical skill?

- Use the Communication Checklist to help you improve and complete Austin's explanation.
- Why is this question a good test of mathematical skill?

Reflecting

- Why is it important for an explanation to be complete and clear?

WORK WITH the Math

EXAMPLE 1 Communicating about powers and exponents

Does $6^2 + 6^5 = 6^7$? Explain.

Austin's Solution

$6^2 + 6^5$ means $(6 \times 6) + (6 \times 6 \times 6 \times 6 \times 6)$.
 6^7 means $6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6$.

I thought about what $6^2 + 6^5$ means and what 6^7 means.



I think $6^2 + 6^5$ and 6^7 are not equal, because 6^7 is 36 times greater than 6^5 , not 36 more.

Check

$$\begin{aligned} 6^2 + 6^5 &= 36 + 6^5 \\ &= 36 + 7776 \\ &= 7812 \\ 6^7 &= 279\,936 \end{aligned}$$

$7812 \neq 279\,936$, so $6^2 + 6^5 \neq 6^7$.

I calculated to make sure. Since powers represent repeated multiplication, I did that before adding.

My answer makes sense, because $36 + 7776$ cannot be the same as 36×7776 .

EXAMPLE 2 Simplifying using order of operations

Calculate $5^2 + [16 \times (2^2 - 6)]$.

Nicole's Solution

$$5^2 + [16 \times (2^2 - 6)]$$

I used order of operations.
I underlined the operations as I did them.

$$\begin{aligned} \underline{5^2} + [16 \times (\underline{2^2} - 6)] & \\ = 25 + [16 \times (4 - 6)] & \\ = 25 + [16 \times (-2)] & \end{aligned}$$

I need to evaluate the expression inside the innermost brackets first. It contains an exponent and so does the first term. I evaluated these powers. I then evaluated the expression in the round brackets by subtracting. This left an expression inside the square brackets which I evaluated by multiplying.

$$\begin{aligned} = 25 + (-32) & \\ = -7 & \end{aligned}$$

I added the remaining numbers.

Communication Tip

You can use the memory aid **BEDMAS** to remember the rules for order of operations. Perform the operations in **B**rackets first.

Calculate **E**xponents and square roots next.

Divide and **M**ultiply from left to right.

Add and **S**ubtract from left to right.

EXAMPLE 3**Simplifying fractions using order of operations**

Calculate $\frac{(4^2 + 3^2) \div 5 + 5}{(4^2 - 3^2) + 3}$.

Derek's Solution

$$\begin{aligned} \frac{(4^2 + 3^2) \div 5 + 5}{(4^2 - 3^2) + 3} &= \frac{(16 + 9) \div 5 + 5}{(16 - 9) + 3} \\ &= \frac{25 \div 5 + 5}{7 + 3} \\ &= \frac{5 + 5}{10} \\ &= \frac{10}{10} \\ &= 1 \end{aligned}$$

I used order of operations to evaluate each expression in the numerator and denominator.

Numerator: I evaluated the expression in the brackets by evaluating the powers then adding. I divided the result by five and then added five to this.

Denominator: I evaluated the expression in the brackets by evaluating the powers then subtracting. I then added three to this.

I divided the numerator by the denominator.

In Summary**Key Idea**

- When everyone follows the same order of operations, everyone gets the same answer to a question.

Need to Know

- Use BEDMAS (Brackets, Exponents, Division, Multiplication, Addition, Subtraction) to remember the order of operations.
- Evaluate the contents in brackets first, starting with the innermost brackets.
- Evaluate powers.
- Multiply and divide from left to right.
- Add and subtract from left to right.

Checking

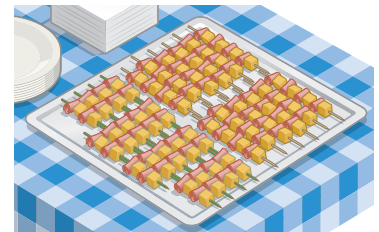
1. Show the steps to evaluate each expression.

a) $4(3)^2$ b) $9^2 + 9 \div 3^2$ c) $12 + (-6)^2 \div 3$

Practising

2. a) Evaluate $3^2 \times 4 + 2^2 - 10$.
 b) Evaluate $3^2 + 4 \times 2^2 - 10$.
 c) Would you use the expression in part a) or part b) for a skill-testing question? Explain why.

3. Evaluate. Explain your strategy.
 a) $3^2 + 3^5$ b) $12^2 + 4^2$ c) $8^4 - 8^2$ d) $7^3 - 2^7$
4. Evaluate.
 a) $4^3 + 3^2 \times 4 \div 2$ c) $\frac{(5^2 - 3) \times 2 \div 11 + 3}{3^2 - (2^2 \times 5)}$
 b) $5^2 - 5 \div 5 + 2 - 1$
5. a) Evaluate $12^2 + 5^2 - 64 \div 4^2$ with a calculator.
 b) Does your calculator follow the order of operations? How do you know?
6. Give an example of a product of two powers that is the same as their sum. Explain how you came up with your example.
7. Explain why 2 is the only base for which $a^2 - a^1 = a^2 \div a^1$.
8. Is it possible for a power with a base of 5 to be equal to a power with a base of 10? Explain.
9. Which is greater: 2^{30} or 3^{20} ? How can you answer this without a calculator?
10. Larry is preparing meat and cheese skewers for a party. He has 18 small skewers and 12 large skewers. Each small skewer needs 2 cubes of cheese. Each large skewer needs twice as many cubes of cheese.
 a) Which expression best describes how many cubes of cheese Larry needs? Explain why.
 A. $18 \times 2^2 + 12 \times 2$ B. $(18 \times 2) + (12 \times 2^2)$
 b) Each small skewer needs 2 cubes of meat. Each large skewer needs double the number of cubes of meat. Write an expression, using powers, to describe how many cubes of meat Larry needs. Explain your answer.
 c) How many cubes of meat does Larry need?



Closing

11. Why is it important to use the order of operations when you use mathematics to communicate?

Extending

12. Ruby copied the solution to a math problem from the board during class. When she got home to review the problem, she spilled her juice on her homework and couldn't make out one of the exponents in the question and part of the solution.
 a) Explain how Ruby can determine the missing exponent.
 b) Rewrite the original question and show all the steps to solve it.
13. To evaluate an expression involving a power, do you have to calculate the power before multiplying and dividing? Explain using examples.

