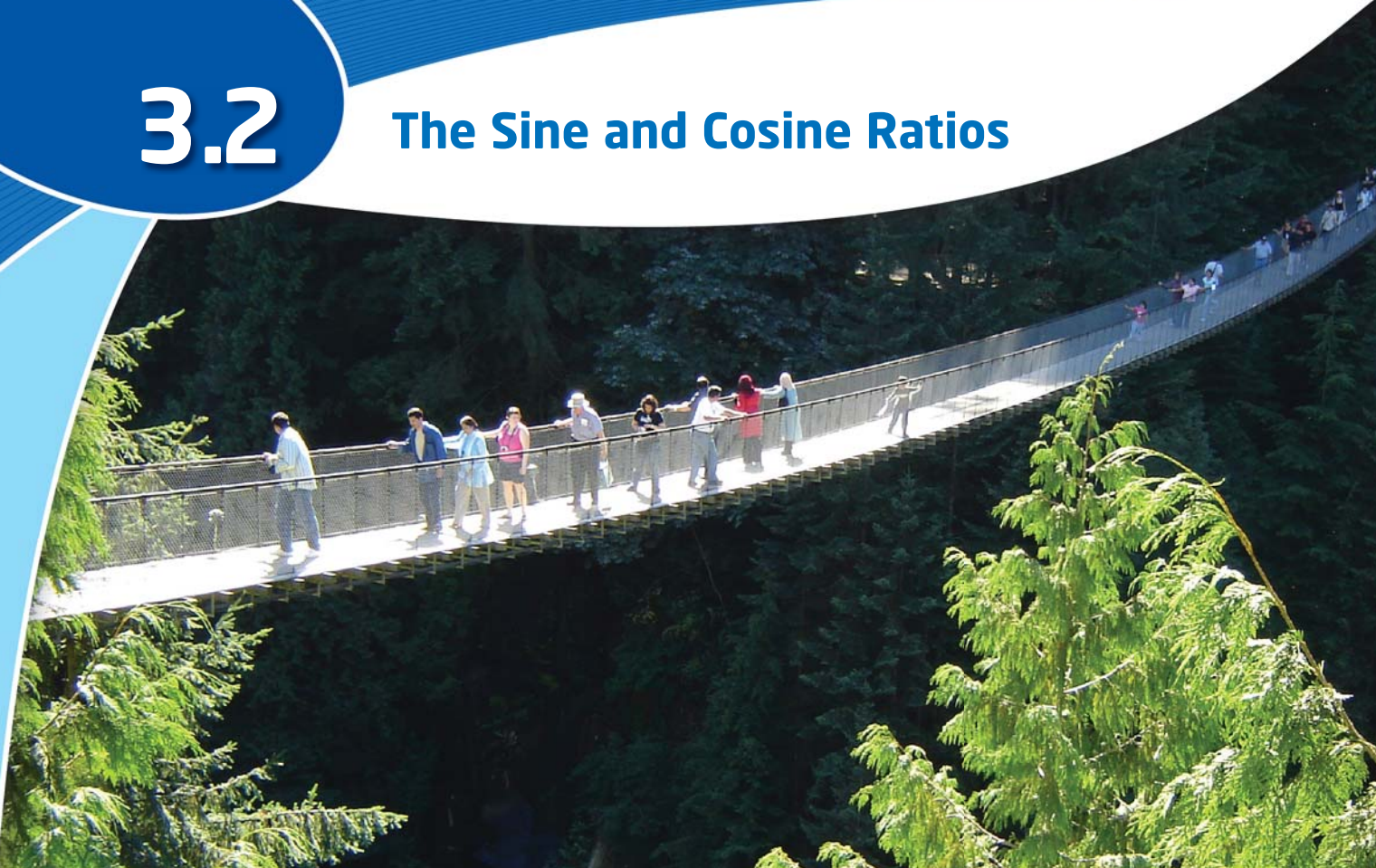


3.2

The Sine and Cosine Ratios



Focus on ...

- using the sine ratio and cosine ratio to solve problems involving right triangles
- solving problems that involve direct and indirect measurement

The first suspension bridge in Vancouver was built in 1889 by George Mackay. He had built a cabin along the canyon wall and needed a bridge to conveniently access his cabin. Mathematical tools, such as trigonometry, can enable you to calculate distances that cannot be measured directly, such as the distance across a river canyon.

In section 3.1, you learned about the tangent ratio. This ratio compares the opposite and adjacent side lengths in reference to an acute angle in a right triangle. There are two other trigonometric ratios that compare the lengths of the sides of a right triangle. These ratios, called the sine ratio and cosine ratio, involve the hypotenuse.

Materials

- protractor
- ruler

Investigate Trigonometric Ratios

1. Choose an angle between 10° and 80° . This will be your reference angle.
2. a) Draw right triangle ABC, using your reference angle.
b) Draw three right triangles similar to $\triangle ABC$ using the same reference angle.

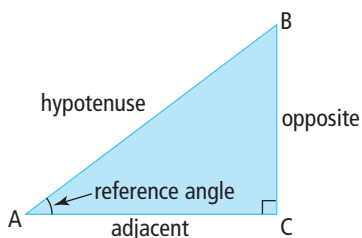
- Write the equivalency statements that show the similarity of each triangle to $\triangle ABC$.
- Label the sides of each triangle. Use the terms hypotenuse, opposite, and adjacent according to the reference angle.
- Measure the sides of each triangle. You may wish to record the measurements in a table similar to this one or using spreadsheet software. Express each ratio to four decimal places.

| Triangle | Length of Hypotenuse | Length of Opposite Side | Length of Adjacent Side | Ratio of Opposite to Adjacent | Ratio of Opposite to Hypotenuse | Ratio of Adjacent to Hypotenuse |
|-------------|----------------------|-------------------------|-------------------------|-------------------------------|---------------------------------|---------------------------------|
| \triangle | | | | | | |

- Complete a similar table using the other acute angle in each triangle as your reference angle.
- Reflect and Respond** Discuss with a partner the results of the calculations of the ratios. Describe any similarities or patterns that you notice.
- What relationships do you observe among the ratios for the angles between the two tables?
- What conclusions can you make about how the ratios relate to your reference angle?

Link the Ideas

The short form for the **sine ratio** of angle A is $\sin A$. The short form for the **cosine ratio** of angle A is $\cos A$.



$$\sin A = \frac{\text{length of side opposite } \angle A}{\text{length of hypotenuse}}$$

$$\cos A = \frac{\text{length of side adjacent to } \angle A}{\text{length of hypotenuse}}$$

sine ratio

- for an acute angle in a right triangle, the ratio of the length of the opposite side to the length of the hypotenuse
- $\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$

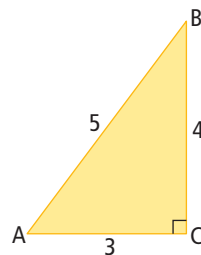
cosine ratio

- for an acute angle in a right triangle, the ratio of the length of the adjacent side to the length of the hypotenuse
- $\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$

Example 1 Write Trigonometric Ratios

Write each trigonometric ratio.

- a) $\sin A$ b) $\cos A$
c) $\sin B$ d) $\cos B$



Solution

a) $\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$

$$\sin A = \frac{BC}{AB}$$

$$\sin A = \frac{4}{5}$$

b) $\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$

$$\cos A = \frac{AC}{AB}$$

$$\cos A = \frac{3}{5}$$

c) $\sin B = \frac{\text{opposite}}{\text{hypotenuse}}$

$$\sin B = \frac{AC}{AB}$$

$$\sin B = \frac{3}{5}$$

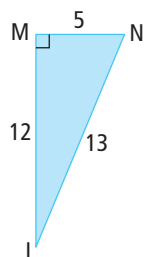
d) $\cos B = \frac{\text{adjacent}}{\text{hypotenuse}}$

$$\cos B = \frac{BC}{AB}$$

$$\cos B = \frac{4}{5}$$

Your Turn

Write each trigonometric ratio.



- a) $\sin L$ b) $\cos N$
c) $\cos L$ d) $\sin N$

Example 2 Evaluate Trigonometric Ratios

The **primary trigonometric ratios** and their inverses can be evaluated using technology.

- a) Evaluate each ratio, to four decimal places.

$$\sin 42^\circ \quad \cos 68^\circ$$

- b) Determine each angle measure, to the nearest degree.

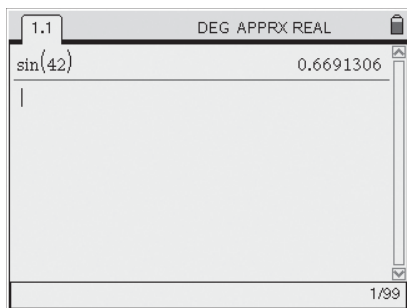
$$\sin \theta = 0.4771 \quad \cos \beta = 0.7225$$

primary trigonometric ratios

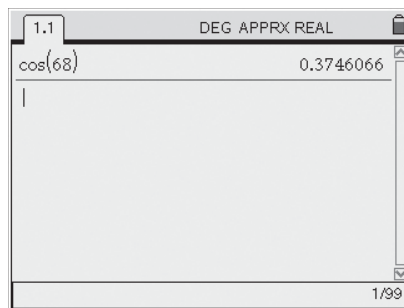
- the three ratios, sine, cosine, and tangent, defined in a right triangle

Solution

a) $\sin 42^\circ \approx 0.6691$



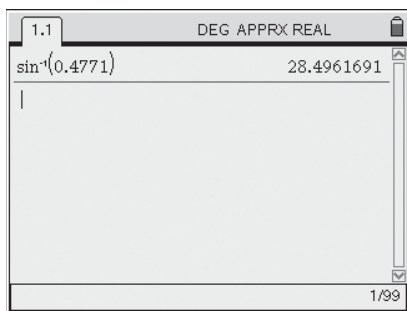
$\cos 68^\circ \approx 0.3746$



b) $\sin \theta = 0.4771$

$$\theta = \sin^{-1}(0.4771)$$

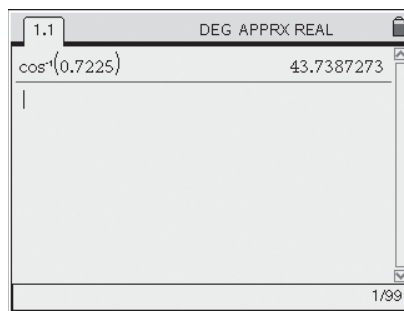
$$\theta \approx 28^\circ$$



$\cos \beta = 0.7225$

$$\beta = \cos^{-1}(0.7225)$$

$$\beta \approx 44^\circ$$



Your Turn

- a) Evaluate each trigonometric ratio, to four decimal places.

$$\sin 60^\circ \quad \sin 30^\circ \quad \cos 45^\circ$$

- b) What is the measure of each angle, to the nearest degree?

$$\sin \beta = 0.4384 \quad \cos \theta = 0.2079$$

Example 3 Determine an Angle Using a Trigonometric Ratio

In the World Cup Downhill held at Panorama Mountain Village in British Columbia, the skiers raced 3514 m down the mountain. If the vertical height of the course was 984 m, determine the average angle of the ski course with the ground. Express your answer to the nearest tenth of a degree.

Solution

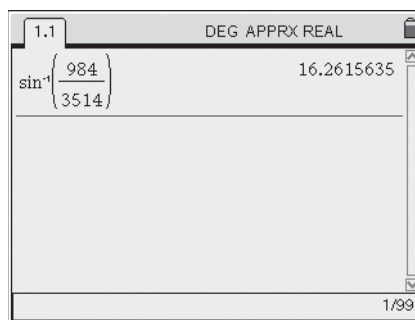
Visualize the problem by sketching a diagram to organize the information.



$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin \theta = \frac{984}{3514}$$
$$\theta = \sin^{-1} \left(\frac{984}{3514} \right)$$
$$\theta = 16.2615\dots^\circ$$

For the unknown angle, the lengths of the opposite side and hypotenuse are known. So, use the sine ratio.



The average angle of the ski course is 16.3° , to the nearest tenth of a degree.

Your Turn

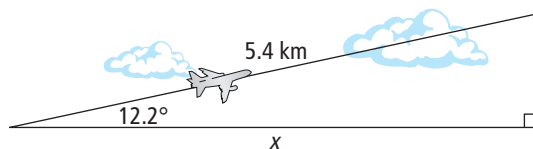
A guy wire supporting a cell tower is 24 m long. If the wire is attached at a height of 17 m up the tower, determine the angle that the guy wire forms with the ground.

Example 4 Determine a Distance Using a Trigonometric Ratio

A pilot starts his takeoff and climbs steadily at an angle of 12.2° . Determine the horizontal distance the plane has travelled when it has climbed 5.4 km along its flight path. Express your answer to the nearest tenth of a kilometre.

Solution

Organize the information by sketching a diagram to illustrate the problem.



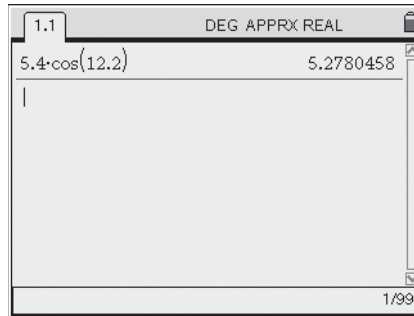
$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos 12.2^\circ = \frac{x}{5.4}$$

$$5.4(\cos 12.2^\circ) = x$$

$$5.278... = x$$

How do you decide which trigonometric ratio to use?



The horizontal distance travelled by the airplane is approximately 5.3 km.

Your Turn

Determine the height of a kite above the ground if the kite string extends 480 m from the ground and makes an angle of 62° with the ground. Express your answer to the nearest tenth of a metre.

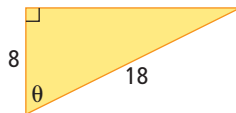
Key Ideas

- The sine ratio and cosine ratio compare the lengths of the legs of a right triangle to the hypotenuse.

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} \quad \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

- The sine and cosine ratios can be used to calculate side lengths and angle measures of right triangles.
- Visualizing the information that you are given and that you need to find is important. It helps you determine which trigonometric ratio to use and whether to use the inverse trigonometric ratio.

Determine the value of θ , to the nearest degree.



$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{8}{18}$$

$$\theta = \cos^{-1}\left(\frac{8}{18}\right)$$

$$\theta = 63.6122...^\circ$$

Angle θ is approximately 64° .

Check Your Understanding

Practise

1. Evaluate each trigonometric ratio to four decimal places.

a) $\cos 34^\circ$

b) $\cos 56.4^\circ$

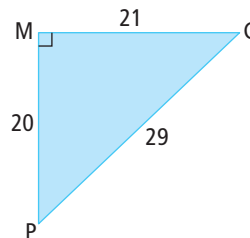
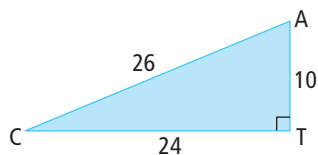
c) $\sin 62.9^\circ$

d) $\sin 19.6^\circ$

e) $\sin 90^\circ$

f) $\cos 80^\circ$

2. Write each trigonometric ratio in lowest terms.



a) $\sin A$

b) $\sin C$

c) $\cos C$

d) $\cos G$

e) $\sin P$

f) $\cos P$

3. Calculate the measure of each angle, to the nearest degree.

a) $\cos A = 0.4621$

b) $\cos \theta = 0.6779$

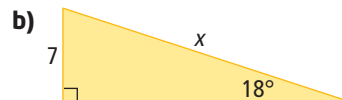
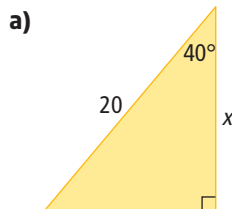
c) $\sin \beta = 0.5543$

d) $\sin C = 1.232$

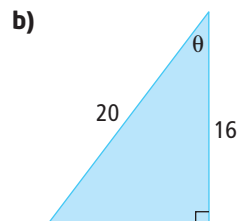
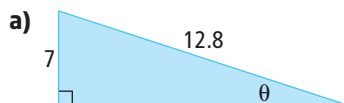
e) $\sin \alpha = \frac{1}{2}$

f) $\cos B = \frac{3}{4}$

4. Determine each length of x . Express your answer to the nearest tenth of a unit.

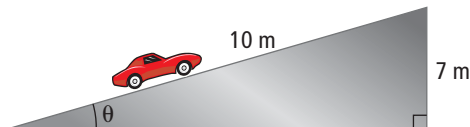


5. Determine the measure of each angle θ . Express your answer to the nearest tenth of a degree.

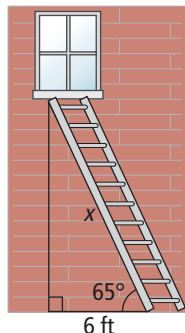


6. Determine the value of each variable. Express each answer to the nearest tenth of a unit.

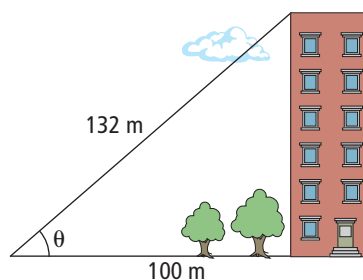
a)



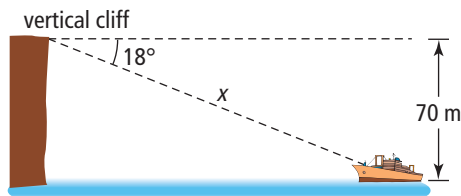
b)



c)

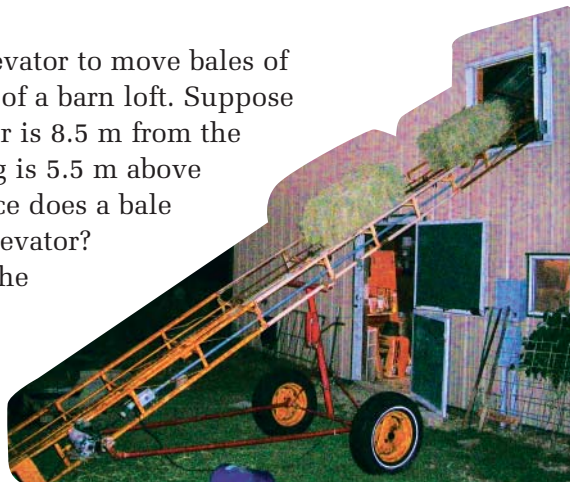


d)



Apply

7. Some farms use a hay elevator to move bales of hay to the second storey of a barn loft. Suppose the bottom of the elevator is 8.5 m from the barn and the loft opening is 5.5 m above the ground. What distance does a bale of hay travel along the elevator? Express your answer to the nearest tenth of a metre.

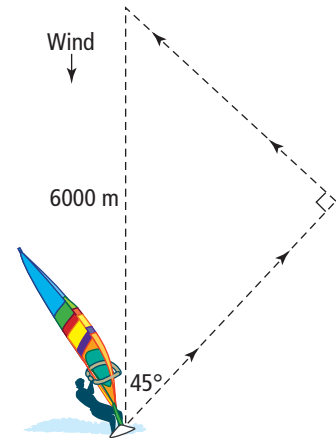


8. A 30-m-long line is used to hold a helium weather balloon. Due to a breeze, the line makes a 75° angle with the ground.
- Draw a right triangle to model the problem. Label the measurements you know. Use variables to represent the unknown measurements.
 - Use trigonometry to determine the height of the balloon. Express your answer to the nearest tenth of a metre.

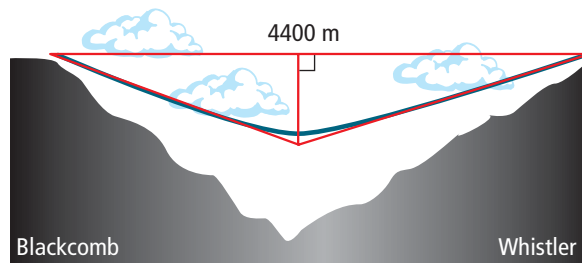


9. Oil rigs are found throughout Alberta. They play a crucial role in the search for crude oil and natural gas products. Determine the height of a rig if a 52-m-long guy wire is attached to the top of the rig and forms an angle of 50° with the ground. Express your answer to the nearest tenth of a metre.

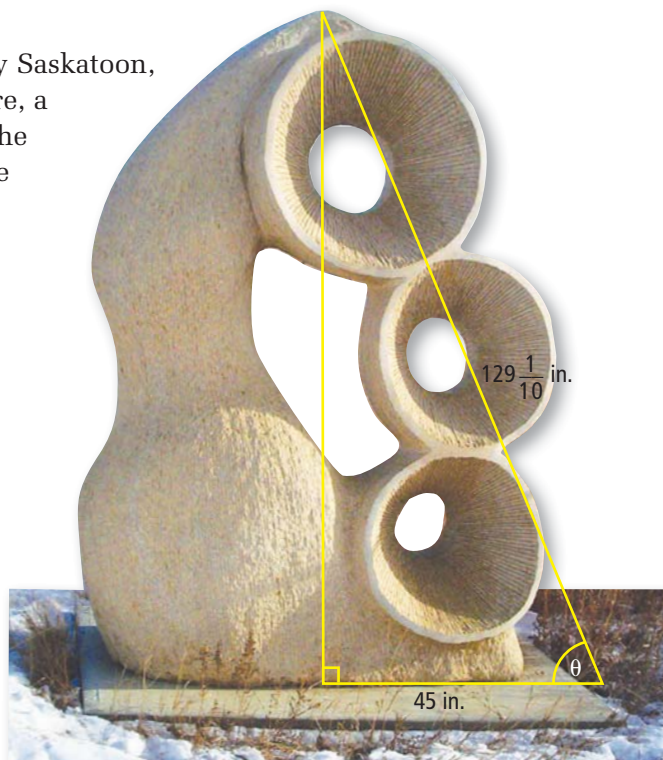
10. Gerry is windsurfing at Squamish Pit, just north of Vancouver, BC. In order to get upwind 6000 m, Gerry sails at a 45° angle to the wind and then turns 90° and heads toward his original destination. How far would he have to sail to get directly upwind the 6000 m? Express your answer to the nearest tenth of a metre.



11. Toonik Tyme is Nunavut's biggest spring festival, celebrating the return of spring. To set up one of the holes for ice golf, the organizers cleared a track in the form of a right angle. The distance from the teeing area to the vertex of the right angle is 180 yd.
- The angle from the teeing area to the flag at the other end of the track is 34° . Draw a diagram of the ice golf hole.
 - Determine the direct distance from the teeing area to the flag, to the nearest yard.
 - How much shorter would the direct distance be than following the track?
12. The PEAK 2 PEAK Gondola connects two mountain ski resorts, Whistler Mountain and Blackcomb Mountain, near Vancouver, BC. The straight-line distance between the two peaks is 4400 m. The gondola travels 4600 m along a cable that sags in the centre. Determine the approximate angle that the cable makes with the horizontal, to the nearest degree.



13. *Dream Maker* is a dolomite sculpture by Saskatoon, SK, artist Floyd Wanner. In the sculpture, a line can be drawn that passes through the centre of the two upper circles. Suppose this line is $129\frac{1}{10}$ in. long and the base line is 45 in. Describe how you might calculate
- the height of *Dream Maker*
 - the angle between the baseline and the line through the two upper circles



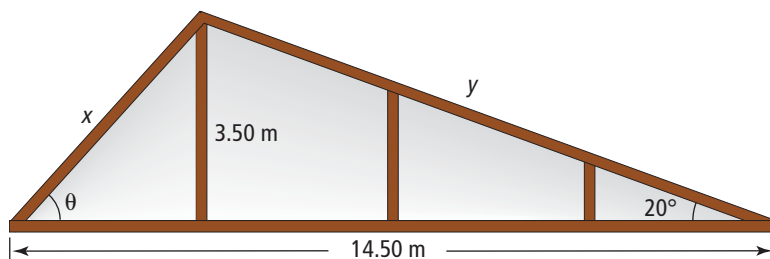
Did You Know?

Dolomite is a rock consisting mainly of calcium and magnesium carbonate. It is mined around the world, including in western Canada. Dolomite is used to improve garden soil. It is also used as an ornamental stone, and in construction materials.

14. At Wapiti Valley Ski Area in Saskatchewan, the beginner slope is inclined at an angle of 11.6° from the horizontal and the advanced slope at an angle of 26.9° from the horizontal.
- Suppose Francis skis 1200 m down the advanced slope while Barbara skis the same distance down the beginner slope. Predict who will cover a greater horizontal distance. Justify your prediction.
 - Calculate the difference between the horizontal distances for the two skiers, to the nearest tenth of a metre.

Extend

15. Michael is building a cabin at Cold Lake, AB. He has drawn a diagram to design his roof truss. Determine the values of x , y , and θ .



16. An equilateral triangle is inscribed in a circle. Determine the side length of the triangle if the diameter of the circle is 200 cm.

Create Connections

Materials

- 1 m of foam pipe insulation, cut lengthwise
- marble or small steel ball
- eight to ten thick books or bricks or a chair
- masking tape
- measuring tape
- table

17. **MINI LAB** Work with a partner or in small groups to explore how varying the angle of a ramp in ski jumping changes the launch angle and duration of flight.



- Step 1** Build a ramp similar to the one shown. Place the edge of the ramp at the end of the table. Make a sketch of the right triangle formed by the pipe insulation, books, and table. Include measurements of the length of each leg of the triangle. Determine the angle formed between the pipe insulation and the table.
- Step 2** Place a marble at the top of the ramp. Without pushing, let it roll. Observe the flight path. Mark the place where the marble first lands on the floor, using masking tape. Repeat this step two more times and record the horizontal distance the marble lands from the edge of the table. You may wish to complete a chart similar to this one.

| Sketch of the Triangle | Measure of the Angle (°) | Distance Measured | | |
|------------------------|--------------------------|-------------------|---------|---------|
| | | Trial 1 | Trial 2 | Trial 3 |
| | | | | |

- Step 3** Adjust the ramp so that it curves downward to the table and runs flat along the table for about 20 cm before it reaches the end. Roll the marble down the track and record the distances.
- Step 4** Add a book to the end of the ramp, so that the ramp curves upward as it nears the end. Roll the marble and record your measurements.
- a) Describe how changing the launch angle of the ramp affects the distance travelled by the marble. Explain why.
 - b) Would changing the angle of the ramp with the table affect the distance the marble travels? Explain.