2.2 Surface Area

Blackfoot Crossing Exhibit Hall

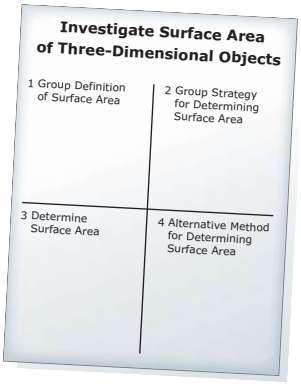
Focus on ...

- solving problems involving the surface area of three-dimensional objects
- finding an unknown dimension of a three-dimensional object given its surface area

Architectural design ideas may evolve from your culture, icons, or everyday life. The exhibit hall at the Blackfoot Crossing Historical Park in southern Alberta is a cultural, educational, and entertainment centre built at the site of the signing of Treaty 7.

The knowledge of surface area of integrated structures is essential when constructing architectural designs like the Blackfoot Exhibit Hall. Architects use surface area to calculate the amount of material needed.

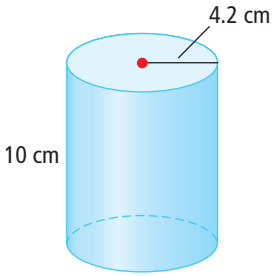
Investigate Surface Area of Three-Dimensional Objects

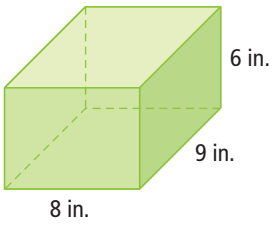


1. As a group, divide a sheet of paper into four quadrants. Label the quadrants with the following:

   Quadrant 1: Group Definition of Surface Area
   Quadrant 2: Group Strategy for Determining Surface Area
   Quadrant 3: Determine Surface Area
   Quadrant 4: Alternative Method for Determining Surface Area

2. As a group, choose one of the three-dimensional objects shown below.





Complete quadrant 1 and one of the three remaining quadrants. Then, pass the sheet on to another group.

3. a) Review and discuss the work of the other groups.

b) Choose and complete one of the remaining quadrants. Then, pass the sheet of paper on to another group.

4. Repeat the process in step 3 for the remaining quadrant.

5. Reflect and Respond

a) Did another group's strategy work better than yours? Why or why not?

b) Did you see any alternative strategies that you preferred? Explain.

c) In quadrant 3, was there a group strategy that you preferred? Explain.

6. Consider the ideas you discussed in step 5. Suggest possible strategies you could use to determine the surface area of each of the following 3-D objects.

a) cone

b) sphere

c) pyramid

WWW Web Link

To learn more about Blackfoot Crossing Historical Park, go to www.mhrmath10.ca and follow the links.

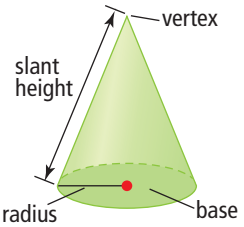
Eagle feather fan over the entrance to Blackfoot Exhibit Hall

Link the Ideas

Surface Area of a Right Cone

cone

- a three-dimensional object with a circular base and a curved lateral surface that extends from the base to the vertex

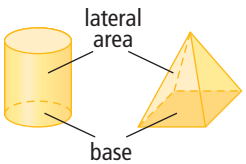


slant height

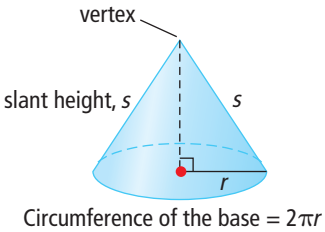
- the shortest lateral distance from the edge of the base of a cone or pyramid to its highest point

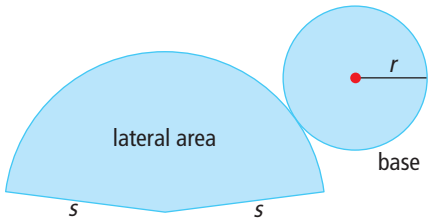
lateral area

- the surface that joins the two bases of a three-dimensional object or that joins the base to the highest point

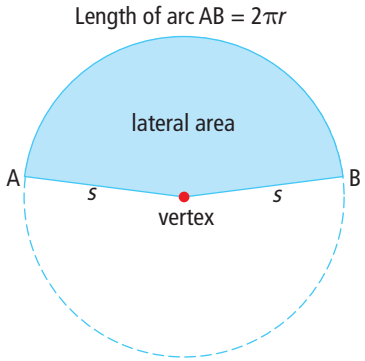


Consider a right **cone** with **slant height** s and base radius r. Construct a net of the cone, including the **lateral area** and the base.





From the lateral area of the net, you can construct a large circle using the vertex of the cone as the centre and the slant height of the cone, s, as the radius.



The lateral area of the base of the cone forms a sector of the large circle.

The circumference of the base of the cone, $2\pi r$, forms the length of the arc AB of the sector.

The circumference of the large circle that is formed from the lateral area is $2\pi s$.

The area of the large circle is πs^2.

To determine the lateral area of the cone you can set up a proportion of corresponding ratios.

$$\frac{\text{lateral area of cone}}{\text{area of large circle}} = \frac{\text{circumference of cone}}{\text{circumference of large circle}}$$

$$\frac{\text{lateral area of cone}}{\pi s^2} = \frac{2\pi r}{2\pi s}$$

$$\frac{\text{lateral area of cone}}{\pi s^2} = \frac{r}{s}$$

$$\text{Lateral area of cone} = \left(\frac{r}{\cancel{s}}\right)(\pi \cancel{s}^{2}{}^{s})$$

$$\text{Lateral area of cone} = \pi rs$$

The lateral area of a right cone with radius r and slant height s is πrs.

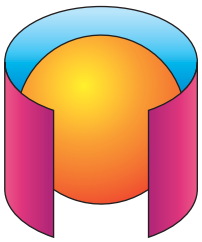
The base of the cone is a circle with radius r, so its area is πr^2.

The total surface area of a right cone is the sum of the areas of the base and the lateral surface.

$$SA_{\text{cone}} = \pi r^2 + \pi rs$$

Surface Area of a Sphere

The formula for the surface area of a **sphere** is linked to the surface area of a right cylinder. Think of wrapping a right cylinder around the sphere.



The diameter of the sphere will be the height of the cylinder.

This height will be $2r$.

The circumference of the sphere will be the circumference of the cylinder. This circumference will be $2\pi r$.

The lateral area of the cylinder, when flattened, forms a rectangle. The area of the rectangle formed by the right cylinder is directly related to the surface area of the sphere.

$A = (length)(width)$

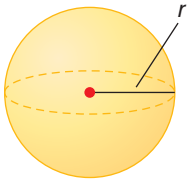
$A = (2\pi r)(2r)$

$A = 4\pi r^2$

$SA_{sphere} = 4\pi r^2$

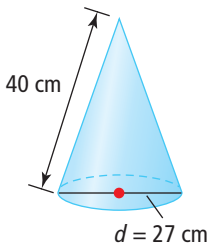
sphere

- a round, ball-shaped object
- a set of points in space that are a given distance (radius) from a fixed point (centre)



Example 1 Calculate the Surface Area of a Right Cone

A right cone has a circular base with diameter 27 cm and slant height 40 cm. Calculate the surface area of the cone, to the nearest tenth of a square centimetre.



Solution

The surface area of a right cone is the sum of the area of the single circular base, B, and the lateral area.

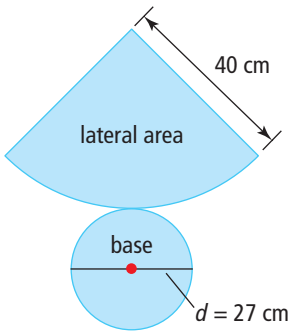
$SA = B + lateral\ area$

$SA = \pi r^2 + \pi rs$

$SA = \pi(13.5)^2 + \pi(13.5)(40)$

$SA = 2269.015\ldots$

Since the diameter of the circle is 27 cm, the radius is 13.5 cm.



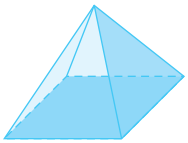
The surface area of the cone is approximately 2269.0 cm^2.

Your Turn

Sketch a right cone with diameter 16 cm and slant height 12 cm. What is its surface area?

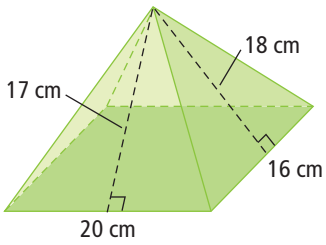
pyramid

- A three-dimensional object with one base and the same number of triangular faces as there are sides on the base



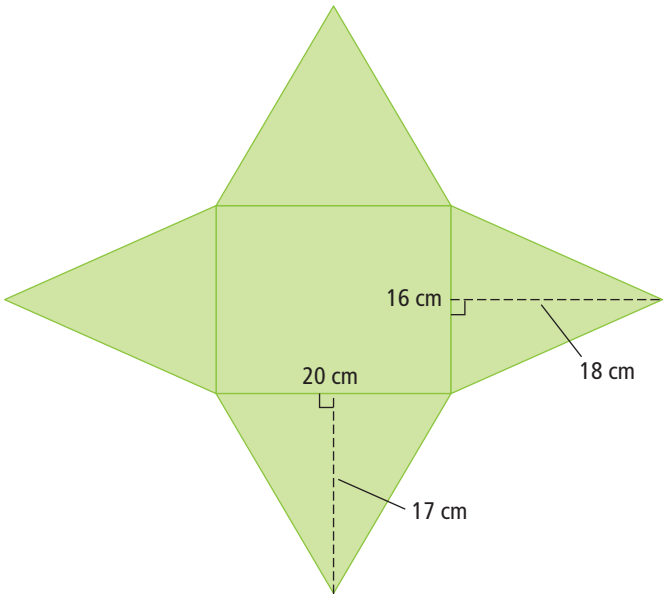
Example 2 Calculate the Surface Area of a Right Pyramid

A right rectangular **pyramid** has a rectangular base measuring 16 cm by 20 cm. The slant height of the triangular face with the shorter base is 18 cm, while the slant height of the triangular face with the longer base is 17 cm. What is the surface area of the pyramid?



Solution

As with a right cone, the surface area of a right pyramid can be calculated as the sum of the area of the base, B, and the lateral area.



$SA = B +$ lateral area

$SA = (\text{length})(\text{width}) + 2[\frac{1}{2}(\text{length})(\text{slant height}_1)] + 2[\frac{1}{2}(\text{width})(\text{slant height}_2)]$

$SA = (20)(16) + 2[0.5(20)(17)] + 2[0.5(16)(18)]$

$SA = 948$

The surface area of the right rectangular pyramid is 948 cm^2.

The lateral area of a right rectangular pyramid is made up of four triangles. The triangles on the opposite faces are the same size.

Your Turn

Sketch a right rectangular pyramid with a square base measuring 10 cm on each side. The slant height of each face is 8.5 cm. What is the surface area of the pyramid?

Example 3 Calculate the Surface Area of a Sphere

A satellite is wrapped with polyester film to protect it during transportation. How much film is required to cover the Echo Satellite that has a circumference of 95.8 m? Express your answer to the nearest tenth of a square metre.

Solution

The formula for the surface area of a sphere is $SA = 4\pi r^2$, where r is the radius. Calculate the radius from the circumference.

$$C = 2\pi r$$
$$95.8 = 2\pi r$$
$$r = \frac{95.8}{2\pi}$$
$$r = 15.247\ldots$$

Substitute the radius into the formula for the surface area.

$$SA = 4\pi r^2$$
$$SA = 4\pi(15.247\ldots)^2$$
$$SA = 2921.333\ldots$$

Approximately 2921.3 m^2 of film is required to cover the satellite.

Your Turn

Find the surface area of a basketball with diameter 23.85 cm. Express your answer to the nearest hundredth of a square centimetre.

Example 4 Determine a Dimension When the Surface Area is Known

The surface area of an official 5-pin bowling ball varies from approximately 459.96 cm^2 to 506.71 cm^2. What is the variation in the diameter of the bowling ball?

Solution

Substitute into $SA = 4\pi r^2$.

$$4\pi r^2 = SA \qquad 4\pi r^2 = SA$$
$$4\pi r^2 = 459.96 \qquad 4\pi r^2 = 506.71$$
$$r^2 = \frac{459.96}{4\pi} \qquad r^2 = \frac{506.71}{4\pi}$$
$$r = \sqrt{\frac{459.96}{4\pi}} \qquad r = \sqrt{\frac{506.71}{4\pi}}$$
$$r \approx 6.05 \qquad r \approx 6.35$$

How do you calculate the diameter from the radius?

The variation in the diameter of an official 5-pin bowling ball is from 12.1 cm to 12.7 cm.

Your Turn

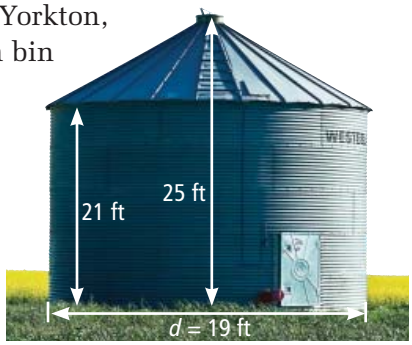
To the nearest millimetre, calculate the radius of a sphere with a surface area of 1 m^2.

Did You Know?

Artifacts from a game similar to bowling were found in the tomb of an ancient Egyptian youth who died in approximately 5200 B.C.E. Ancient Polynesians rolled stones at objects from a distance of 60 ft (18.29 m). This is the same distance as from the foul line to the headpin in modern-day bowling.

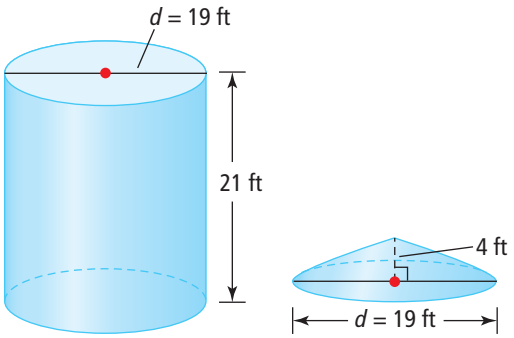
Example 5 Visualize and Find Surface Areas of Composite Objects

A farm equipment manufacturer in Yorkton, SK, has decided to construct a grain bin using galvanized steel. How much steel is required to construct the grain bin as shown? Express your answer to the nearest hundredth of a square foot. Do not include overlap of the steel sheets where they are fastened together.



Solution

Visualize the cylindrical base with a right cone-shaped roof. Sketch the two parts of the structure and add appropriate dimensions.



Remember that the bottom and top of the cylindrical portion and the bottom of the conical roof are not included.

surface area of cylindrical shape $= \pi d \times height$

$SA_{cylinder} = \pi(19)(21)$

$SA_{cylinder} = 1253.495\ldots$

The circumference of a circle can be found using the formula πd or $2\pi r$.

To calculate the surface area of the cone, you need to know the slant height first.

Use the Pythagorean relationship.

$s = \sqrt{4^2 + 9.5^2}$

$s = \sqrt{16 + 90.25}$

$s = \sqrt{106.25}$

$s = 10.307\ldots$

surface area of conical shape $= \pi rs$

$SA_{cone} = \pi(9.5)(10.307\ldots)$

$SA_{cone} = 307.636\ldots$

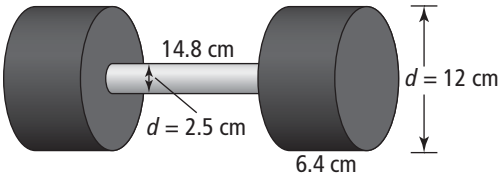
The combined surface area is $1253.495\ldots + 307.636\ldots = 1561.132\ldots$

The total surface area of the grain bin, to the nearest hundredth of a square foot, is 1561.13 ft^2.

You will need a minimum of 1562 ft^2 of galvanized steel to construct the grain bin.

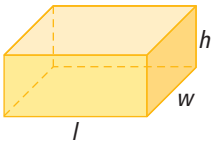
Your Turn

Calculate the surface area of the following composite object, to the nearest tenth of a square centimetre.

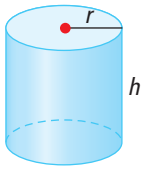


Key Ideas

- The surface area of a right cylinder and of a right prism can be calculated using the area of the bases (top and bottom) plus the lateral area.

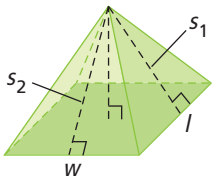


$SA_{\text{prism}} = 2lw + 2lh + 2wh$

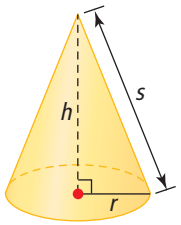


$SA_{\text{cylinder}} = 2(\pi r^2) + 2\pi rh$

- The surface area of a right pyramid and of a right cone can be calculated using the area of the base plus the lateral area.

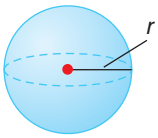


$SA_{\text{pyramid}} = lw + 2[\frac{1}{2}ls_1] + 2[\frac{1}{2}ws_2]$



$SA_{\text{cone}} = \pi r^2 + \pi rs$

- The surface area of a sphere depends on the radius only.



$SA_{\text{sphere}} = 4\pi r^2$

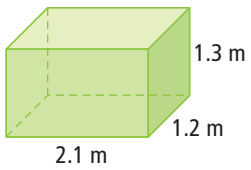
Check Your Understanding

Practise

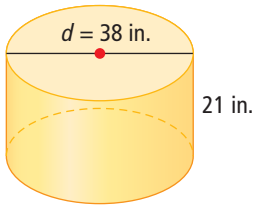
For each of the following, express your answers to the nearest tenth of a unit where necessary.

1. Calculate the surface area of each of the following.

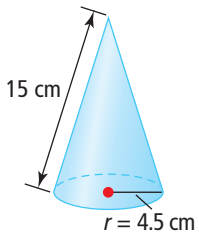
a)



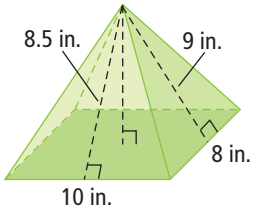
b)



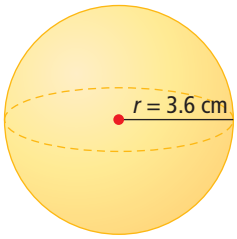
c)



d)



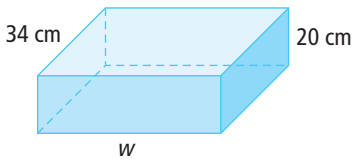
e)



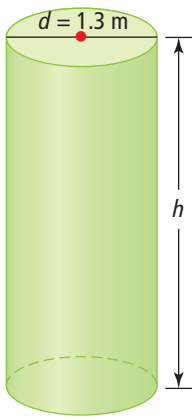
2. Sketch a right pyramid with a square base measuring 16 ft by 16 ft. The slant height is 12 ft. What is the surface area of the pyramid?

3. For each of the following, the surface area is given. Calculate the missing dimension.

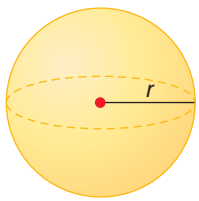
a) $SA = 6020 \text{ cm}^2$



b) $SA = 47.4 \text{ m}^2$

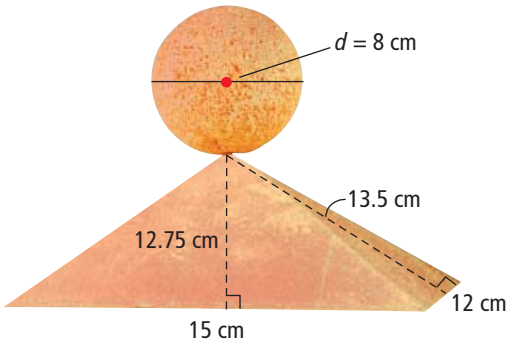


c) $SA = 91.4 \text{ m}^2$



4. A closed box has a surface area of 126 in.^{2}. The base of the box is 5 in. by 3 in. Sketch a diagram and find the height of the box.

5. Calculate the surface area of this object composed of a pyramid and a sphere.



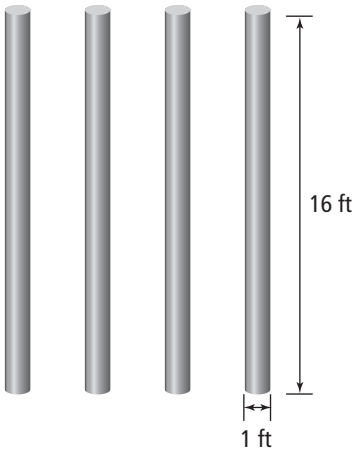
Apply

6. Austin is helping to build the set for a school play. There are four cylindrical pillars standing on the stage that need to be painted. Each pillar is 16 ft high and 1 ft in diameter, as shown below. Austin calculated the surface area to the nearest hundredth of a square foot. His work is shown below.

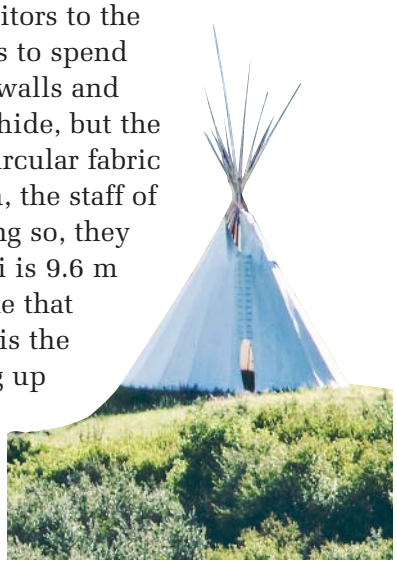
$SA = 2(\pi r^2) + \pi d(h)$
$SA = 2(\pi)(0.5)^2 + \pi(1)(16)$
$SA = 0.5\pi + 16\pi$
$SA = 16.5\pi$
$SA \approx 51.84$
There are four cylinders, so
$SA = 4 \times 51.84$
$SA = 207.36$

The surface area is 207.36 ft^2.

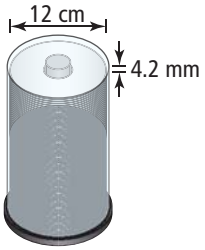
Austin thinks he made an error in his work. Discuss whether Austin actually made an error. What surface area would you paint?



7. One of the activities available to visitors to the Blackfoot Crossing Historical Park is to spend the night in a tipi. Historically, the walls and floor of a tipi were made of buffalo hide, but the current tipi is canvas with a large circular fabric floor. At the beginning of the season, the staff of the park set up the tipis. While doing so, they determine that the diameter of a tipi is 9.6 m and its slant height is 7.3 m. Assume that the tipi approximates a cone. What is the minimum amount of canvas making up the sides of the tipi, to the nearest hundredth of a square metre? Do not include any seam allowances.



8. **Unit Project** Compact discs are sometimes packaged in cylindrical stacks of 100. Each CD has a thickness of 1.2 mm and a diameter of 12 cm.



a) The outside radius of the storage case is 0.7 cm more than that of the CD. The height of the case is 4.2 mm more than that of the stack of 100 CDs. What is the surface area of the storage case, excluding the base, to the nearest square centimetre?

b) If a rectangular CD jewel case holding a single CD is 0.5 cm wider than the CD, 2.5 cm longer than the CD, and 8 times the thickness of the CD, what is the surface area of the jewel case?

9. A designer is working on a line of athletic equipment. He is designing a cylindrical punching bag, similar to the one shown. The designer wants to use a maximum of 1.3 m^2 of material to make the bag, and has determined that the diameter of the bag should be 36 cm. Determine the maximum possible height of the bag, to the nearest tenth of a centimetre.

10. Earth has a diameter of approximately 8000 mi. Land forms about 29% of the surface area of Earth. Assume Earth is a sphere. Estimate the area of land on Earth.

11. The photo shows a traditional Haida hand drum that has a diameter of $14\frac{7}{8}$ in. and is 3 in. deep. What is the minimum amount of hide used to make the drum if the hide covers only the top and lateral surfaces? Express your answer to the nearest square inch.

Traditional Haida hand drum showing twin salmon transforming into the next generation. Traditional Haida hand drums are used in ceremony, cultural events such as potlatches, and as artwork. Traditional drums should always be handled with respect following appropriate protocol.

12. The Muttart Conservatory in Edmonton began with a donation from the Muttart family. There are four greenhouses, each in the shape of a right pyramid with a square base. Each of the two largest greenhouses has a base that measures 26 m on each side and has a slant height of 35.4 m. How much glass is needed for each large greenhouse? Express your answer to the nearest square metre.

13. What is the surface area of a glass bead that has a diameter of 11 mm? Express your answer to the nearest square millimetre.

14. Photographers often use a light tent to get the best lighting for items they photograph for museums, catalogues, or online sales. This tent is cylindrical with a conical roof. The diameter of the tent is 1 m, the height of the tent is 190 cm, and the cylindrical wall of the tent is 135 cm high. What is the surface area of the light tent, to the nearest square metre?

Did You Know?

Around the year 1148, the French played *le paume*, meaning "the palm of the hand." This game developed into *jeu de paume*, *real tennis*, *royal tennis*, or, simply *tennis*.

15. Squash is a racquet sport played with a small, soft ball. The game is named for the fact that it is very easy to "squash" the ball. The rules of the game state that the ball must have a diameter of 40 mm ± 0.5 mm. This means that the actual diameter can be up to 0.5 mm more or less than 40 mm.

a) What is the minimum and maximum surface area of a regulation squash ball? Express your answers to the nearest square millimetre.

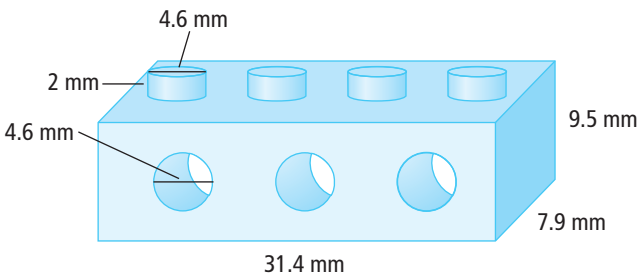
b) Squash balls are often packaged in cubical boxes as shown in the photo. If the box is to be sized so that the ball fits exactly in the box, find the minimum and maximum dimensions of the box.

Extend

For each of the following, express your answers to the nearest tenth of a unit where necessary.

16. Gas stations often have a supply of small paper funnels that customers can use to add oil to their vehicle engines without spilling. Each funnel is a right cone with a small hole cut out of the top. The funnel has a slant height of 13.2 cm and the diameter of the large opening is 9.5 cm. The diameter of the small opening is 1.5 cm. Determine the amount of paper in the funnel.

17. A building block is shown below.



The base of the block measures 31.4 mm by 7.9 mm, and it has a height of 9.5 mm. Each right cylinder on top of the block has a diameter of 4.6 mm and a height of 2 mm. Each hole in the block has a diameter of 4.6 mm. Determine the surface area of the block that needs to be painted if you do not paint the bottom of the block.

18. Use spreadsheet software to help investigate how changing the radius of a sphere changes its surface area. Create a spreadsheet like the one shown below.

	A	B	C	D
1	Investigating Changes in Dimensions of a Sphere			
2	Stretch Factor	Radius	Surface Area	Ratio of New SA to Original SA
3	1	2	50.265	1
4	2	4		
5	3			
6	4			
7	5			

a) Use spreadsheet formulas to complete the spreadsheet. Depending on your software, you may need to type "PI" or "PI()" for π. See your spreadsheet's help feature if you need assistance.

b) Compare the stretch factor for the radius to the ratio of the new surface area to the original surface area. What pattern do you notice?

c) Use your pattern to predict the surface area of the sphere if you multiply the radius by 6. Extend your spreadsheet to check your answer.

d) In your own words, express the relationship between a change in the radius of a sphere and the change in its surface area.

Create Connections

19. Sketch an example of a composite object with dimensions in centimetres. Explain how you can find the surface area. Describe how you would convert your answer to an appropriate imperial unit of area.

20. People communicate in different ways, such as orally, in writing, and even using sign language. In your own words, explain why the surface area of any object is expressed in square units.

Did You Know?

American Sign Language is the most commonly used form of sign language in North America.