2.1 Units of Area and Volume

Focus on ...

- solving problems that involve area and volume units within SI and imperial systems
- using mental math to judge the reasonableness of a solution to a problem

Throughout the history of recorded sound, new technologies have been introduced to replace older ones. One example is the progression from sheet music to vinyl records to cassette tapes to CDs to MP3s. As the music industry has advanced, many outdated products have been abandoned. However, there are many music enthusiasts who still prefer the vinyl record over its digital replacements. They believe that vinyl records have a warmer sound than CDs or MP3s. Vinyl records also feature large album covers with imaginative graphics, pullout photos, and liner notes.

Investigate Units of Area

Unit Project

1. By the 1880s, wax cylinders were used to record music. Sound was recorded in the grooves on the outside of hollow cylinders of slightly softened wax. Standard cylinders were about 4 in. long with a diameter of $2\frac{1}{4}$ in. One cylinder could play about two minutes of music or other sound.

a) Calculate the outside surface area of one of these hollow cylinders.

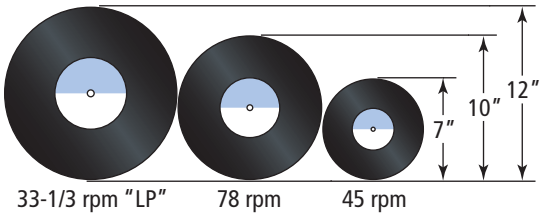
b) Calculate the rate of the area needed to record the music to the number of minutes of music.

2. In the 1930s, RCA produced the LP. These vinyl records were pressed on a 30 cm diameter flexible plastic disc. Each LP could hold about 45 min of music using both sides.

a) Calculate the circular area of both sides of an LP.

b) Calculate the rate of the area needed to record the music to the number of minutes of music.

3. Records evolved into three sizes and three forms of sound reproduction, the 45 rpm, the 78 rpm, and the $33\frac{1}{3}$ rpm (or LP). Newly pressed records were inserted in a paper or cellophane envelope or sleeve, and then slipped into a printed record jacket or album cover.



a) Choose one size of vinyl record. Calculate the area of a record jacket needed for this vinyl record.

b) Design an album cover that you would use for the record jacket for your favourite recording artist.

4. Reflect and Respond

a) List possible advantages and disadvantages of vinyl records compared to wax cylinders for recorded music.

b) Vinyl records have recently made a comeback and sales are on the increase. Discuss with a partner some possible reasons for this increase in popularity.

c) Brainstorm other advancements in music storage since the early wax cylinders and discuss how technology has changed the storage of music.

WWW Web Link

To learn more about how vinyl records are made, go to www.mhrmath10.ca and follow the links.

Link the Ideas

Sometimes you may know the dimensions in one measurement system, but need to determine the area or volume in the other measurement system. To work with units of area and volume in both measurement systems, you need to understand the relationships between the units of length in each system. Remember that area involves square units while volume involves cubic units.

Example 1 Convert Between SI Units for Area

Cochrane Mural

An art class is creating a mural mosaic. They know that future art classes will have to contribute to the project to complete it. Each person in the class makes a painting on a 15-cm by 15-cm panel. Then, all the panels are assembled into the mural mosaic. There are 25 students in the art class, so the panels they create will be assembled into a square with each side containing five panels. What area is required for this part of the mural?

Solution

Two students calculate the area required for this part of the mural.

Cassy calculates the area of each individual panel.

$A = s^2$

$A = (15)^2$

$A = 225$

Each panel is 225 cm^2.

There are 25 panels.

$(25)(225) = 5625$

The total area is 5625 cm^2.

How does Cassy use her knowledge of the area of a square?

Stefan thinks of one large rectangle. Since there are five panels on each side, each measuring 15 cm long, he concludes that each side of the rectangle will measure 75 cm. He converts to 0.75 m and then calculates the area:

$A = lw$

$A = (0.75)(0.75)$

$A = 0.5625$

The area is 0.5625 m^2.

How does Stefan use his knowledge of the area of a rectangle?

Both students calculated the correct area. When they compared answers, they realized that to convert between square metres and square centimetres, you need to multiply or divide by 10 000.

$(1 \text{ m})(1 \text{ m}) = 1 \text{ m}^2$

$(100 \text{ cm})(100 \text{ cm}) = 10\,000 \text{ cm}^2$

So, $1 \text{ m}^2 = 10\,000 \text{ cm}^2$

Recall that there are 100 cm in 1 m.

Your Turn

a) Determine the area of a rectangle that is 1.7 m by 2.5 m, in square centimetres.

b) Determine the area of a rectangle that is 50 mm by 25 mm, in square metres.

Example 2 Work With Units for Area

Tiles imported from different countries sometimes have imperial dimensions. A tile layer may need to convert from square feet to square centimetres.

One type of floor tiles is sold in squares measuring 1 ft by 1 ft.

The abbreviation for inches is in. or ″.
The abbreviation for feet is ft or ′.
The abbreviation for yards is yd.

a) What is the area of one tile in square centimetres?

b) The tile layer is working with an area that measures 8 ft by 4 ft. What is the area, to the nearest hundredth of a square centimetre? to the nearest square metre?

Solution

a) Calculate the area of one tile.

$A = s^2$
$A = (30.48)^2$
$A = 929.0304$

Recall that 1 ft = 30.48 cm.

The area of one tile is 929.03 cm², to the nearest hundredth of a square centimetre. You can conclude that 1 ft² is approximately 929.03 cm².

1 ft ≈ 30 cm
A tile that is 1 ft by 1 ft is approximately 30 cm by 30 cm.
(30)(30) = 900
The area of one tile is about 900 cm².

b) The tile layer needs to convert the dimensions of 8 ft by 4 ft to SI units.

The length is 8 ft.
8(30.48) = 243.84
Therefore, the length is 243.84 cm.

The width is 4 ft.
4(30.48) = 121.92
Therefore, the width is 121.92 cm.

How could you use the value for 8 ft in centimetres to determine the value for 4 ft in centimetres?

$A = lw$
$A = (243.84)(121.92)$
$A = 29\ 728.9728$
The tile area is approximately 29 728.97 cm².

Since 100 cm = 1 m, then 10 000 cm² = 1 m².
To express the area in square metres, divide by 10 000.

$$\frac{29\ 728.9728}{10\ 000} = 2.97289728$$

The tile area is approximately 3 m².

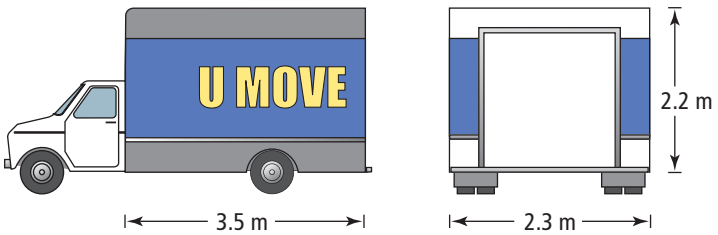
Your Turn

a) Determine the area of a rectangle that is 10 cm by 100 cm, in square feet.

b) Determine the area of a rectangle that is 6 in. by 4 in., in square millimetres.

Example 3 Work With Units for Volume

Sahid has numerous boxes to load onto a moving truck.



What is the volume of the truck, to the nearest cubic foot?

Solution

Convert the dimensions to feet.

The length is 3.5 m.
$3.5(3.281) \approx 11.4835$ 1 m ≈ 3.281 ft
Therefore, the length is approximately 11.4835 ft.

The width is 2.3 m.
$2.3(3.281) \approx 7.5463$
Therefore, the width is approximately 7.5463 ft.

The height is 2.2 m.
$2.2(3.281) \approx 7.2182$
Therefore, the height is approximately 7.2182 ft.

Find the volume of the truck by using the formula $V = lwh$.
$V = lwh$
$V \approx (11.4835)(7.5463)(7.2182)$
$V \approx 625.514\,314$

How does Sahid use his knowledge of the volume of a rectangular prism?

The volume of the truck is approximately 626 ft^3.

Your Turn

Convert the volume of an object that measures 3 cm by 4 cm by 10 cm to cubic inches.

1 cm ≈ 0.3937 in.

Key Ideas

Proportional reasoning can be used to

- solve problems involving area or volume units within SI
- solve problems involving area or volume units within the imperial system
- solve problems requiring the conversion of area or volume within and between the SI and imperial systems using linear dimensions

Convert 0.62 m^2 to square centimetres.

1 m = 100 cm

$1\ m^2 = (100\ cm)(100\ cm)$

$= 10\ 000\ cm^2$

$0.62\ m^2 = (0.62)(10\ 000\ cm^2)$

$= 6200\ cm^2$

0.62 m^2 is equal to 6200 cm^2.

Calculate the volume of a rectangular prism with dimensions 1 ft by 3 ft by 5 ft in cubic metres.

1 ft = 0.3048 m 3 ft = 3(0.3048 m) = 0.9144 m 5 ft = 5(0.3048 m) = 1.524 m

$V = (0.3048\ m)(0.9144\ m)(1.524\ m)$

$V = 0.4247526989\ m^3$

The volume of the prism is approximately 0.42 m^3.

Check Your Understanding

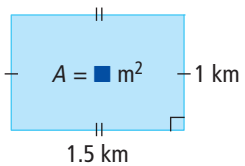
Practise

For help with #1 and #2, you may need to refer to the table of conversion factors.

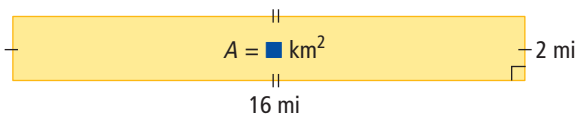
Imperial Unit	SI Unit
1 in.	2.54 cm
1 ft	0.3048 m
1 yd	0.9144 m
1 mi	1.609 km

1. Calculate the following areas to the indicated SI unit. Express your answers to the nearest tenth of a square unit.

a)



b)

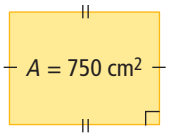


c) the area of a rectangle 35 in. by 10 in., in square centimetres

d) the area of a rectangle 21 ft by 50 ft, in square metres

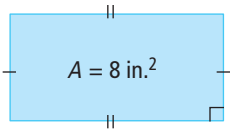
2. Determine possible dimensions for each area. Then, use your dimensions to calculate the area to the indicated equivalent.

a)



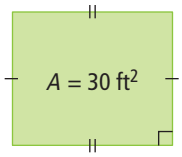
$A =$ ■ mm^2

b)



$A =$ ■ m^2

c)



$A =$ ■ cm^2

3. At the *Festival du Voyageur*, an outdoor winter beaverball court is being built. The court measures 9 m by 18 m. Your task as a volunteer is to cover it with a tarp. You find that tarps are sold by the square yard. Find the area required for the tarp in square yards.

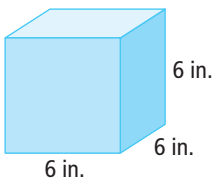
Festival du Voyageur, Winnipeg

Did You Know?

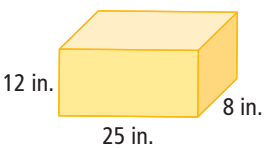
The *Festival du Voyageur* was founded in 1969 by a group of Saint-Boniface entrepreneurs. Originally a 3-day event held in Winnipeg's French Quarter, this event has evolved into a 10-day province-wide celebration. The *Festival du Voyageur* celebrates the *joie de vivre* of the fur traders, who established the Red River Colony and the growing French-Canadian community in western Canada. Held every February, the *Festival du Voyageur's* emphasis is on the beauty of winter, with numerous historical, educational, and entertaining activities.

4. For each of the following containers, find the volume in cubic centimetres.

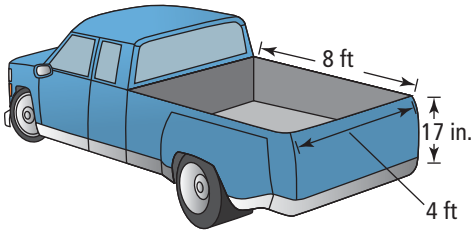
a) a cube with sides measuring 6 in.



b) a right rectangular prism with sides measuring 12 in. by 8 in. by 25 in.



5. A pickup truck has a box that measures 4 ft by 8 ft by 17 in. What is the volume of the box to the nearest tenth of a cubic metre?



Apply

6. The *Quilt of Belonging* is a project created by artist Esther Bryan. The quilt includes 263 handmade, 11-in.-square blocks, representing each immigrant, First Nation, Métis, and Inuit group in Canada. Seventy of these blocks each represent a First Nation, Métis, or Inuit group in Canada. What is the total area of these 70 blocks, in square feet and in square metres?

Did You Know?

The *Quilt of Belonging* is "... the largest and most inclusive work of textile art made about Canada." Esther Bryan, the project artist, says, "The completed quilt, with its many parts, shows that we all can be integrated into the fabric of Canada, living together harmoniously, learning to respect one another for our differences while celebrating what we have in common."

7. Gentry's family is buying a new home. Gentry is concerned about the size of his bedroom in the new home. Gentry's current bedroom measures 10′6″ by 11′3″. The floor plan shows that his new bedroom would have an area of 11.4 m^2.

a) Which room is bigger? by what percent?

b) The carpet in Gentry's new bedroom is worn and needs to be replaced. He finds new carpet he likes for $12.99 per square yard. What is the cost of the new carpet for Gentry's new room, before tax?

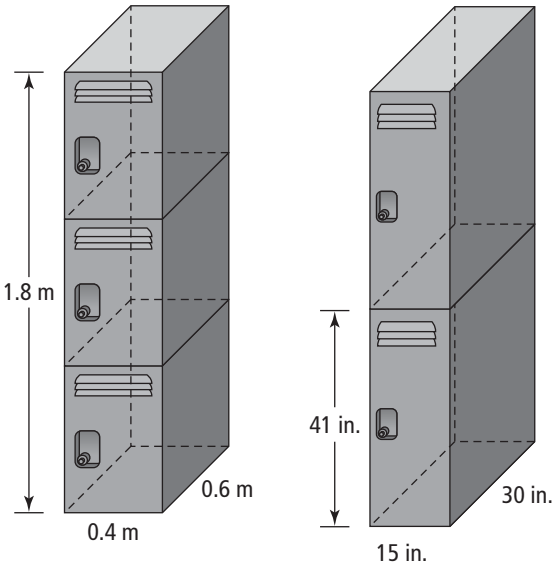
8. Andrea is preparing an installation manual for a cell-phone tower to be used in a European country. The tower specifications are in imperial units, and she must convert them to SI for her client. The specifications state that the signal for the cell-phone tower covers a circular area of radius 2.5 mi. What is this area in SI units?

9. A tile layer has an entranceway to tile. The entrance measures 5′2″ by 3′6″. The tiles each measure 4″ by 4″.

a) What is the area of the entranceway, in square inches?

b) When working on tiling projects, it is recommended that the installer purchase 10% extra material. How many tiles should be purchased for this project?

10. At the local recreation centre, you have a choice between two different types of lockers. You can choose a single locker from a double stack, or a single locker from a triple stack. The dimensions of each stack of lockers are shown below.



a) Which type of locker would give you more space?

b) How much more space, in cubic metres, would you have?

11. Describe how you would determine each volume in the indicated units. What is the volume?

a) a cube with dimensions 1 m by 1 m by 1 m in cubic centimetres

b) a rectangular prism with dimensions 5 cm by 7.1 cm by 10 cm in cubic metres

c) a rectangular prism with dimensions 0.5 m by 1 m by 5 m in cubic millimetres

Extend

12. In the imperial system, large areas of land are measured in acres. For example, the area of Shaw Millennium Skate Park in Calgary, the biggest public skate park in North America, is about 1.85 acres.

a) One acre is the same as 43 560 ft^2. What is the area of Shaw Millennium Skate Park in square feet?

b) Show how you would determine the area you found in part a) in square metres.

c) The SI unit for large areas is the hectare. One hectare is the same as 10 000 m^2. Express the area of Shaw Millennium Park in hectares.

13. The maps in an atlas use scale diagrams. Choose a rectangular area on a map of northern Canada. Using the scale from the atlas, calculate the area of your rectangle in square miles and in square kilometres.

Shaw Millennium Skate Park, Calgary

Create Connections

14. What occupations do you think would use both the imperial and SI measurement systems?

15. Describe your preferred method for converting between SI and imperial measures. Why did you choose this method?

16. Think of your local grocery store. In which departments of the store would you need measurement conversions? Explain.