

# 1.3

## Converting Between SI and Imperial Systems



### Focus on ...

- comparing SI and imperial units using referents
- solving problems that involve conversion of linear measurements between SI and imperial systems
- using mental mathematics to confirm the reasonableness of a solution to a conversion problem

Some people use music to express their thoughts through song lyrics. Many song lyrics tell a story.

- Name some lyrics or song titles that include imperial measurements, such as miles, feet, or miles per hour.
- What SI measurements are comparable to those measurements?
- Describe how the converted measurements could change the song.

### Materials

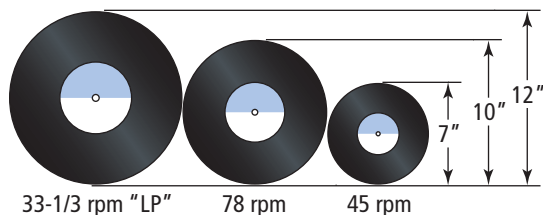
- compact disc (CD)

## Investigate Relationships Between SI and Imperial Measurements

### Unit Project

- a) Use a referent to estimate the diameter of a CD in SI units. How did you make your estimate?
  - b) Use a referent to estimate the diameter of a CD in imperial units.
2. Measure the diameter of a CD, to the nearest millimetre.

3. Calculate the diameter of each vinyl record in SI units.



4. a) Calculate the circumference of a CD and each of the three vinyl records shown, in SI units.

b) Compare the sizes of the four recording devices.

### 5. Reflect and Respond

a) Discuss with a partner. Which recording device is able to store more music—a vinyl record or a CD?

b) Develop a method you could use to compare the amount of music that each device stores to the size of the device. Test your method in both imperial and SI units.

c) Describe how you think laser technology has affected the storage of music. Support your answer with information you have gathered in your research.

### WWW Web Link

To learn more about laser technology, go to [www.mhrmath10.ca](http://www.mhrmath10.ca) and follow the links.

## Link the Ideas

To convert from one measurement system to another, you need to understand the relationships between the units of length in each system. Conversions involve proportional reasoning and unit analysis.

Conversions between measurement systems may be approximate or exact. The imperial yard has been defined as 0.9144 m. This would be considered an exact conversion.

Since  $1 \text{ yd} = 0.9144 \text{ m}$  and  $1 \text{ yd} = 3 \text{ ft}$ ,  $3 \text{ ft} = 0.9144 \text{ m}$ .

$$1 \text{ ft} = 1 \text{ ft} \left( \frac{0.9144 \text{ m}}{3 \text{ ft}} \right)$$

$$1 \text{ ft} = 0.3048 \text{ m}$$

To convert from metres to yards, 1 m is often given as 1.094 yd. Verify whether this conversion is exact or approximate.

Let  $x$  represent the number of yards in 1 m.

$$0.9144 \text{ m} = 1 \text{ yd}$$

$$\frac{0.9144 \text{ m}}{1 \text{ yd}} = \frac{1 \text{ m}}{x \text{ yd}}$$

$$x = \frac{1}{0.9144}$$

$$x = 1.0936\dots$$

The conversion  $1 \text{ m} = 1.094 \text{ yd}$  is approximate.

The following are some common conversions.

**Exact Conversions**

$$1 \text{ in.} = 2.54 \text{ cm} \qquad 1 \text{ ft} = 30.48 \text{ cm} \qquad 1 \text{ yd} = 0.9144 \text{ m}$$

**Approximate Conversions**

$$\begin{array}{lll} 1 \text{ mm} \approx 0.0394 \text{ in.} & 1 \text{ cm} \approx 0.3937 \text{ in.} & 1 \text{ m} \approx 1.094 \text{ yd} \\ 1 \text{ m} \approx 3.281 \text{ ft} & 1 \text{ km} \approx 0.6214 \text{ mi} & 1 \text{ mi} \approx 1.609 \text{ km} \end{array}$$

**Example 1 Convert Between SI and Imperial Units for Length**

Researchers at the Harvard-Smithsonian Center for Astrophysics made an announcement in January, 2001. They stated that they had “frozen light” by using super-cooled vapour to slow the speed of light waves to zero. The speed of light in a vacuum is defined as 299 792 458 m/s.

- a) Estimate the speed of light in miles per second.
- b) Predict whether the actual speed of light is greater than your estimate. Justify your prediction.
- c) Calculate the answer, to the nearest mile per second.

**Solution**

- a) Since there are 1000 m in 1 km, 299 792 458 m/s can be converted to 299 792.458 km/s or approximately 299 792 km/s.

$$299\,792 \frac{\text{km}}{\text{s}} \left( \frac{1 \text{ mi}}{1.6 \text{ km}} \right) \approx 187\,370 \frac{\text{mi}}{\text{s}} \qquad \text{Recall that } 1 \text{ mi} \approx 1.6 \text{ km.}$$

The speed of light can be estimated as 187 370 mi/s.

- b) The actual speed of light, in miles per second, is less than the estimate because the SI speed was rounded down before converting.

c) Use unit analysis.  $1 \text{ mi} = 1760 \text{ yd} \left( \frac{0.9144 \text{ m}}{1 \text{ yd}} \right)$

$$1 \text{ mi} = 1609.344 \text{ m}$$

$$299\,792\,458 \frac{\text{m}}{\text{s}} \left( \frac{1 \text{ mi}}{1609.344 \text{ m}} \right) = 186\,282.397 \frac{\text{mi}}{\text{s}} \qquad \text{Why is the speed of light used here in metres per second?}$$

The speed of light is approximately 186 282 mi/s.

**Your Turn**

Swimmer Brian Johns of Richmond, BC, represented Canada at the 2008 Olympics in Beijing. He finished 7th in a race that one news report referred to as 400 m long and another news report referred to as  $\frac{1}{4}$  mi. Are the two measurements equivalent? If not, which distance do you think is more accurate? Justify your reasoning.

## Example 2 Solve a Problem Involving Linear Measurements



Your class needs to lay mats on the gymnasium floor for a gymnastics meet. The gym measures 84' by 50'. A scale drawing of one mat is shown. The scale is 1 : 30.5.

A classmate thinks that 131 mats are needed. Do you agree? Explain.



### Solution

The mat in the diagram measures 8 cm by 4 cm.

Use the given scale.

$$\begin{aligned} \text{length of mat} &= 8(30.5) & \text{width of mat} &= 4(30.5) \\ l &= 244 & w &= 122 \end{aligned}$$

The length of the mat is 244 cm and the width is 122 cm.

#### Length of mat

$$244 \text{ cm} \left( \frac{1''}{2.54 \text{ cm}} \right) \approx 96''$$

#### Width of mat

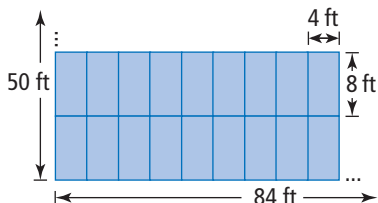
$$122 \text{ cm} \left( \frac{1''}{2.54 \text{ cm}} \right) \approx 48''$$

There is a 2:1 ratio of length to width in the measurements of the mat. The SI-to-imperial conversions confirm a 2:1 ratio of length to width.



The dimensions of the mat are 8 ft by 4 ft. Sketch a layout of the mats in the gym.

**How do you know the dimensions are 8 ft by 4 ft?**



Suppose you do not visualize the mats and you only work with areas.

$$\frac{\text{area of gym}}{\text{area of mat}} = \frac{84(50)}{8(4)}$$

Therefore, about 131 mats would be needed.

The width of each mat is 4 ft.

$$\frac{84}{4} = 21$$

So, there are 21 columns of mats.

The length of each mat is 8 ft.

$$\frac{50}{8} = 6\frac{1}{4}$$

So, there are 6 rows of mats and some uncovered space.

To cover the gym floor, there are 21 columns and 6 rows of mats.

$$21(6) = 126$$

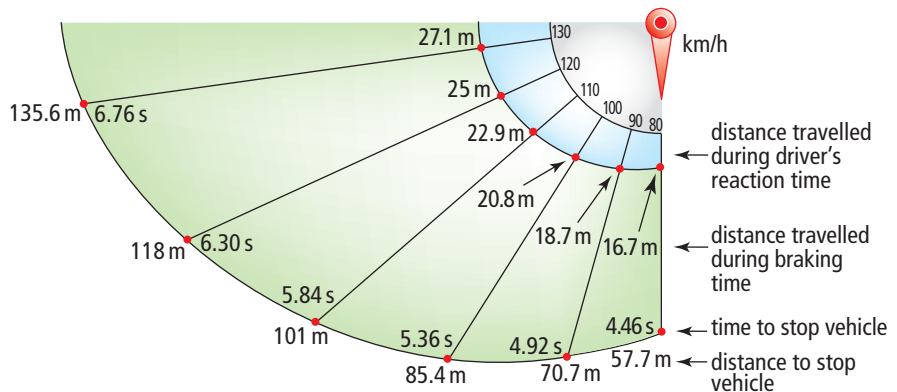
Therefore, 126 mats are needed to cover most of the gym floor.

### Your Turn

- How many paving stones measuring  $7\frac{1}{2}$  in. by  $7\frac{1}{2}$  in. are needed to cover an area that is 1 yd by 1 yd?
- To tile a floor that is 3 m by 4 m, how many tiles measuring 30 cm by 50 cm would you buy? Add 10% extra tiles for areas that require tiles to be cut.

### Example 3 Determine Stopping Distances

The distance required to stop a moving vehicle is the sum of the distances travelled during the reaction time and the braking time. The diagram shows the theoretical stopping distance at various speeds.



- What factors might affect the reaction time and braking distances?
- If a vehicle is travelling at 100 km/h, approximately what distance is travelled while the brakes are being applied?
- Convert 55 mph into kilometres per hour. What is the approximate stopping distance when a vehicle is travelling at this speed? Express your answer in feet.

### Solution

- a) Factors that affect reaction time may include being tired, distraction by passengers, time in the vehicle, loud music, or reduced visibility due to weather or dirty windows.

Factors that affect braking distance may include condition of brakes, condition and/or type of tires, speed, type of road surface, outside temperature, or road conditions (dry, icy, wet).

- b) Find 100 km/h on the diagram.

The reaction-time distance at 100 km/h is 20.8 m.

The total stopping distance at 100 km/h is 85.4 m.

Total stopping distance = reaction-time distance + braking distance

$$85.4 = 20.8 + b$$

$$85.4 - 20.8 = b$$

$$64.6 = b$$

If travelling at 100 km/h, a vehicle travels approximately 65 m while the brakes are being applied.

Subtract the distances represented by the line segments on the diagram.  
Stopping distance  $\approx 85 - 20$  or 65



- c) Convert 55 miles to kilometres.

Use proportional reasoning.

Recall that 1 mi  $\approx$  1.609 km.

Let  $x$  represent the number of kilometres.

$$\frac{1 \text{ mi}}{55 \text{ mi}} \approx \frac{1.609 \text{ km}}{x \text{ km}}$$

$$x \approx (55)1.609$$

$$x \approx 88.495$$

Therefore, 55 mph converts to approximately 88 km/h.

From the diagram, when a vehicle is travelling at 90 km/h, the stopping distance is 70.7 m.

The speed 88 km/h is just under 90 km/h, so convert 70 m to feet.

Use the conversion  $0.9144 \text{ m} = 1 \text{ yd}$  or 3 ft.

$$70 \text{ m} \left( \frac{3 \text{ ft}}{0.9144 \text{ m}} \right) = 229.658... \text{ ft}$$

When travelling at a speed of 55 mph, a vehicle takes approximately 230 feet to stop.

### Your Turn

- a) Use the diagram from Example 3 on page 40. Determine the difference between the reaction-time distances for speeds of 110 km/h and 120 km/h. Express your answer in feet.
- b) Convert 90 km/h into miles per hour. What is the approximate stopping distance for a vehicle travelling at this speed? Express your answer in yards.

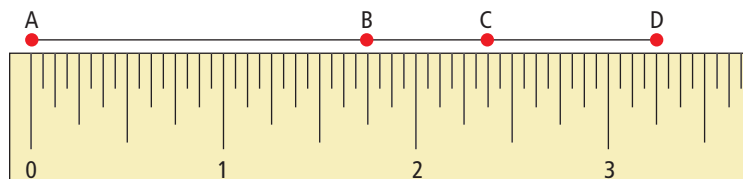
## Key Ideas

- When solving problems involving measurement, it is crucial to work with the same units. You may need to convert units within one measurement system (for example, inches to feet) or between imperial and SI units.
- If an exact conversion between systems is required, use  $1 \text{ yd} = 0.9144 \text{ m}$  to find a conversion between the required units.  
 $1 \text{ yd} = 0.9144 \text{ m}$   
 $36 \text{ in.} = 0.9144 \text{ m}$       **Divide both sides by 36.**  
 $1 \text{ in.} = 0.0254 \text{ m}$   
 $1 \text{ in.} = 2.54 \text{ cm}$       **This is an exact conversion**
- Sometimes you use approximate values, such as  $1 \text{ in.} \approx 2.5 \text{ cm}$  or  $1.6 \text{ km} \approx 1 \text{ mi}$  when estimating between measurement systems.

## Check Your Understanding

### Practise

1. Use the diagram of an imperial ruler to help answer the questions below.



- a) What is the length of AC? Give your answer as a fraction.
  - b) Suppose you replace the imperial ruler with an SI ruler. What is the length of AD, in millimetres?
  - c) What is the difference in the lengths of segments AB and CD? Give your answer in SI and imperial units.
2. Convert each measurement to the unit specified.
    - a) The diameter of a human hair is 0.001 in. (hundredth of a millimetre)
    - b) On an NBA basketball court, the width of the key (painted area beneath the basket) is 4.9 m. (feet)
    - c) A snowmobile trail in Alberta is 26 mi 385 yd long. (hundredth of a kilometre)
    - d) An envelope has a height of  $3\frac{7}{8}$  in. (hundredth of a centimetre)

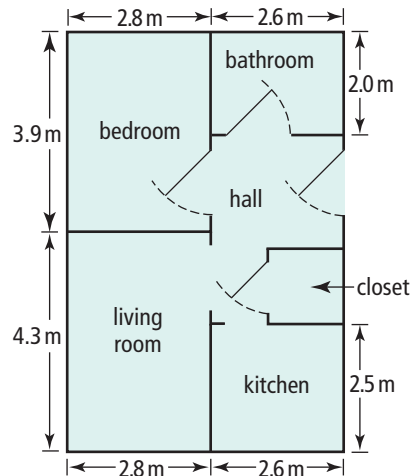
3. People have always used parts of the body for measuring length. For example, you may have heard the measurements of a horse stated by the number of “hands.” Work with a partner. State your answers in both SI and imperial units.

- Estimate and then measure your hand span.
- How many of your hand spans are needed to measure the length of your desk?
- Estimate and then measure your pace. This is the distance between your heels when you take a step while walking. To be more accurate, you may wish to measure 10 paces and calculate the average.
- How many of your paces would it take to walk a mile? a kilometre?
- Which measurement system do you prefer to use when estimating? Why?



4. The floor plan for a one-bedroom apartment is shown.

- If the closet is square, estimate the length of one of its sides.
- Determine the scale of the floor plan.
- Calculate the length of one side of the closet. Express your answer to the nearest tenth of a metre.
- What are the dimensions of the bedroom, in imperial units?



5. Read the following paragraph about the Columbia Icefield. Convert each SI measurement to an equivalent imperial measurement.

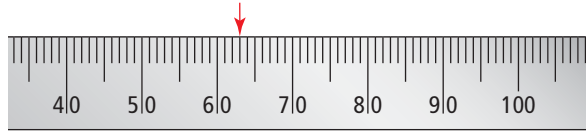
The Columbia Icefield straddles the boundary between Alberta and British Columbia. It is the largest glacier in North America, south of the Arctic Circle. It has a maximum depth of 365 m. Its highest points are Mount Columbia, at 3745 m, and Mount Athabasca, at 3491 m. The average snowfall across the icefield is 100 cm per year.



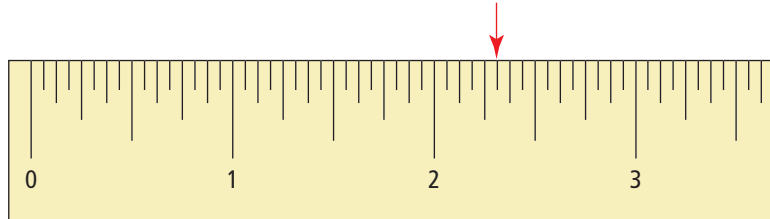


6. What is the reading represented on each measuring device? Estimate and then calculate each equivalent measurement in the other system (SI or imperial).

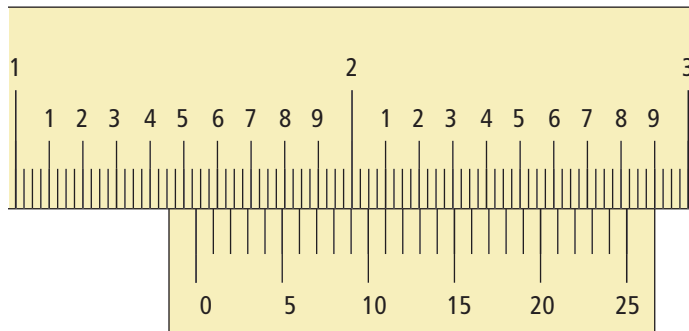
a) SI ruler



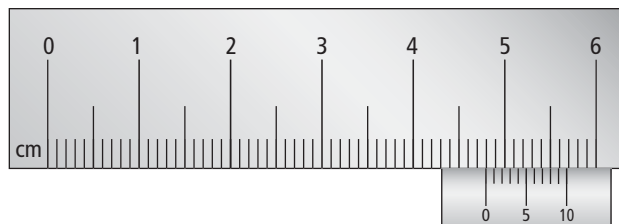
b) imperial ruler



c) imperial caliper



d) SI caliper



### Apply

7. A traditional Inuit dog sled uses teams of Qimmiq or sled dogs on separate lines. The lines are fastened directly to the komatik or sled. Each dog has a harness with an average length of  $3\frac{1}{2}$  ft. Suppose a dog sled uses a team of 13 dogs.
- Calculate the approximate total length of rope needed to harness the team.
  - Calculate the total length of the harness in SI units. Explain why you chose this particular unit.

8. Alex is on his way home to Moosomin, SK. He was visiting friends in Mohall, North Dakota. As soon as he gets on the highway, he sees the sign to the right.



As Alex passes the Canada-U.S. border, he sees this sign. What is the driving distance between Mohall and Moosomin, in SI units?



9. a) Discuss with a classmate your preferred method of converting the following units.
- metres to kilometres
  - metres to centimetres
  - yards to miles
- b) Develop a strategy for converting from a smaller unit of length to a larger unit. Does your strategy apply in both SI and imperial systems of measurement? Explain.
- c) How might your strategy change when you are converting from a larger unit of length to a smaller one?
10. Margaux and Penny each travel with their families to St. Pierre-Jolys, MB, for the annual Frog Follies. When the girls arrive, they compare how far they travelled from their homes.
- Margaux lives in Arborg, MB, and travelled 164 km.
  - Penny lives in Grand Forks, North Dakota, and travelled 113 mi.
- Penny is not familiar with SI distances. Explain how to determine the approximate conversion from kilometres to miles. Who travelled farther? Explain.
11. The deepest lake in the world is Lake Baikal, in Russia. It has a depth of 5369 ft. Canada's deepest lake is Great Slave Lake, named after the Slavey (Dene Tha) peoples who live there. It is located in the Northwest Territories and has a depth of 2015 ft. Quesnel Lake, south of Prince George, BC, is the deepest fjord lake in the world. It has a depth of 506 m.
- Compare the depth of Quesnel Lake with the depths of Lake Baikal and Great Slave Lake. Give each answer as a decimal, to the nearest metre.

### Did You Know?

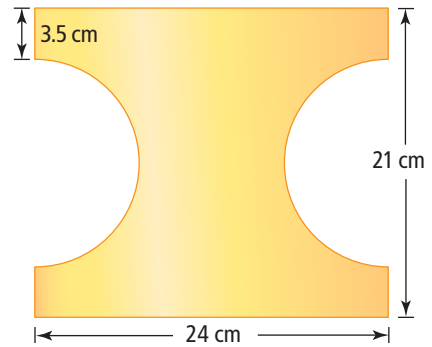
Canada has two Niagara Falls. Most people know about Niagara Falls in Ontario. The other Niagara Falls is on Quesnel Lake in British Columbia. At the mouth of Niagara Creek, water falls over 200 ft into the east arm of Quesnel Lake.

### Did You Know?

The pull of gravity is greater closer to the centre of Earth. For this reason, you would weigh about 2.2 lb more at the North Pole than on the equator.

12. Earth is not a perfect sphere. It flattens slightly at the poles. A person standing at the North Pole is about 13 mi closer to the centre of Earth than a person standing on the equator. How far would you be from the centre of Earth when standing on the North Pole? Give your answer in miles. Assume the equatorial radius of Earth is 6380 km.

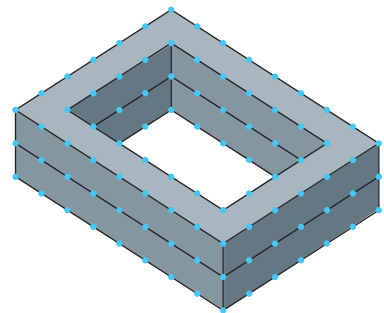
13. a) Calculate the perimeter of the figure shown. Express your answer to the nearest quarter of an inch.
- b) Discuss your steps with a partner. Then, describe what you think is the easiest way to calculate the perimeter of the figure in inches.



14. **(Unit Project)** One type of 80 GB MP3 player has dimensions of 4.14 cm (width) by 9.15 cm (height) by 0.85 cm (thickness). The storage capacity is about 20 000 songs. Each LP vinyl record holds an average of 12 songs and is approximately  $\frac{1}{9}$  in. thick.
- a) Calculate the number of LPs you would need to store as many songs as the MP3 player. Use mental mathematics to show that your answer is reasonable.
- b) Suppose you stack the LPs. Calculate the height of the stack. Compare it with the height of the MP3 player. Give your answer as a ratio in lowest terms.

### Extend

15. A farmer wants to build a wall using concrete blocks. She draws a diagram showing the wall and identifies the corners of the “cubes.” Each concrete block is the same size as two cubes.



- a) How many concrete blocks does the farmer need?
- b) Each concrete block is 20 cm by 20 cm by 40 cm. What are the outside dimensions of the walled enclosure, in inches?
- c) The farmer wants to build a new wall to fit tightly around the outside of the first one. How many concrete blocks are needed? Explain how to get the answer from your answer to part a).

16. Winnipeg's *River Arch* spans 23 m along the Trans-Canada Highway. The approximate area under a symmetrical arch can be calculated using the formula  $A = \frac{w(H + 4h)}{6}$ .

In the formula,

- $A$  represents the area, in square metres
- $w$  represents the width of the arch, in metres
- $H$  represents the height of the centre of the arch, in metres
- $h$  represents the height of the arch measured one quarter of the distance from each end of the arch, in metres



- a) Assume the arch is symmetrical. Calculate the approximate area under *River Arch*, in square metres.
- b) Would this formula work for imperial units? Explain.

### Create Connections

17. a) State three distances that are often measured in SI units and three distances that are often measured in imperial units.
- b) For each distance in part a), indicate whether it is more appropriate to use an exact measurement or an approximate measurement. Justify your reasoning.
18. Using the conversion factor  $1 \text{ yd} = 0.9144 \text{ m}$ , show how to convert from a small imperial unit to a larger SI unit.

### Did You Know?

*River Arch* symbolizes the past and present of Winnipeg. The arch contains images of bison, wheat, and ploughed fields. *River Arch* is located where two bridges cross the Red River in downtown Winnipeg.