1.2 Imperial Measurement

Focus on …
- providing referents for linear measurements
- describing a strategy for taking a linear measurement
- solving problems that involve linear measurement using instruments
- estimating linear measurements

Most of the world uses the SI measurement system. The United States uses the **imperial system** for linear measurement. This system is based on the older English units of measurement derived from nature and everyday activities.

Canada began a transition from the imperial system to SI in 1970, but imperial measurement is still used. Why do you think this is true? Where have you seen imperial units used to measure distance? What other imperial units can you identify?

The Royal Observatory in Greenwich, United Kingdom displays metal representations for the smaller imperial distance measurements. These include the inch, foot, and yard.

imperial system
- a system of measurement based on British units

Materials
- imperial linear measuring instrument

Investigate Referents for Imperial Measurement

One of the smallest imperial units for measuring distance is the inch. The next unit larger than the inch is the foot. The next unit larger than the foot is the yard. Work with a partner. Share your answers with your classmates.

1. Identify the length of one inch, one foot, and one yard on your measuring instrument.

a) How many inches are in one foot?

b) How many feet are in one yard?

c) How many inches are in one yard?

2. List objects at school or at home that you could use as a referent for one inch, one foot, and one yard.

3. What could you use as a referent for one mile?

4. **Reflect and Respond**
   a) Choose an object in your classroom. Describe how to use a referent to measure the dimensions of the object.
   b) Explain why you chose your referent.

Link the Ideas

The following units are the basic imperial units used for measuring distances. They are in order from smallest to largest. The abbreviations and symbol are in brackets.

inch (in. or ″)	
foot (ft or ′)	1 ft = 12 in.
yard (yd)	1 yd = 3 ft or 36 in.
mile (mi)	1 mi = 1760 yd or 5280 ft

How many inches are in $3\frac{1}{2}$ yd?

$$1 \text{ yd} = 36 \text{ in.}$$
$$3\frac{1}{2} \text{ yd} = 3\frac{1}{2} \text{ yd}\left(\frac{36 \text{ in.}}{1 \text{ yd}}\right)$$
$$3\frac{1}{2} \text{ yd} = 126 \text{ in.}$$

There are 126 in. in $3\frac{1}{2}$ yd.

ME

$\left(3\frac{1}{2}\right)(36) \approx 4(30)$ or $3(40)$.
So, an estimate is 120.

Approximately how many miles are in 12 640 ft?

$$1 \text{ mi} = 5280 \text{ ft}$$
$$\frac{1 \text{ mi}}{5280 \text{ ft}} = \frac{x \text{ mi}}{12\,640 \text{ ft}}$$
$$\frac{1(12\,640)}{5280} = x$$
$$2.3939\ldots = x$$

There are approximately 2 mi in 12 640 ft.

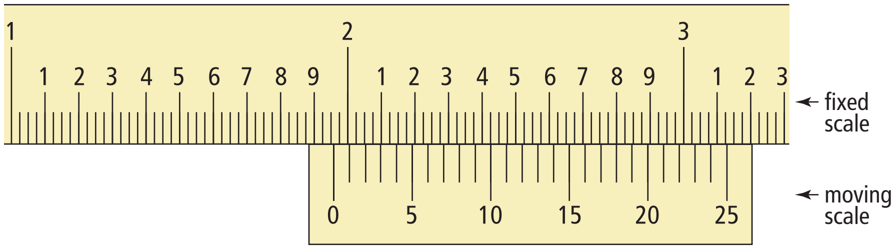
Measuring Instruments

Different measuring devices are used depending on the precision required. An imperial ruler or measuring tape can measure distances to the nearest $\frac{1}{16}$ in. A caliper can measure to the nearest $\frac{1}{1000}$ in.

WWW Web Link

To watch a video showing how to read an imperial caliper, go to www.mhrmath10.ca and follow the links.

Follow these steps to read an imperial caliper.



1. Read the whole number and tenth values on the fixed scale.

This reading is 1.9 in.

2. Determine where zero on the moving scale lies relative to, in this case, the 9 on the fixed scale.

It is 2 small divisions beyond the 9.

$\frac{2}{4}$ of $\frac{1}{10} = \frac{1}{20}$ or 0.05.

This reading is 0.05 in.

3. Identify the next line on the moving scale that aligns with a line on the fixed scale. In this example, it is 10.

This reading is 0.010 in.

4. Add the measurement readings from steps 1 to 3.

The final reading is 1.960 in. $(1.9 + 0.05 + 0.010 = 1.960)$

Example 1 Determine Imperial Distances

The photograph shows a polar bear near Churchill, MB. The scale of the photograph is 1 : 24.

a) Calculate the height of the bear's back, to the nearest inch.

b) What is the length of the bear? State your answer in feet and whole inches.

Solution

a) Use an instrument to measure the distance from the highest point on the bear's back to the ground.
height of bear in photo = 2 in.

Let x represent the height of the actual bear.

$$\text{Scale} = \frac{\text{distance on photograph}}{\text{actual distance on ground}}$$

$$\frac{1}{24} = \frac{2}{x}$$

What measurement unit will the actual height be in? How do you know?

$$\frac{1}{24}(24x) = \frac{2}{x}(24x)$$

$$x = 48$$

The height of the bear is 48 in.

b) Measure the distance from the bear's nose to the rear leg.
length of bear in photo = $3\frac{3}{8}$ in.

Let z represent the actual length of the bear.

$$\frac{1}{24} = \frac{3\frac{3}{8}}{z}$$

$$\frac{1}{24} = \frac{\frac{27}{8}}{z}$$

$$z = 24\left(\frac{27}{8}\right)$$

$$z = 81$$

The length of the bear is 81 in.

Convert 81 in. to feet and inches. To do this calculate the number of whole feet and then find the number of inches remaining.

$$\frac{81 \text{ in.}}{12} = 6 \text{ ft } 9 \text{ in.}$$

Use 1 ft = 12 in. to find the number of whole feet.

The number of inches in 6 ft is 6(12 in.) = 72 in.

Remainder = 81 − 72
= 9

The length of the bear is 6 ft 9 in.

Your Turn

The photograph of a muskox uses a scale of 1:30. Calculate the height of the muskox and the distance between the tips of its horns. State each answer in feet and inches.

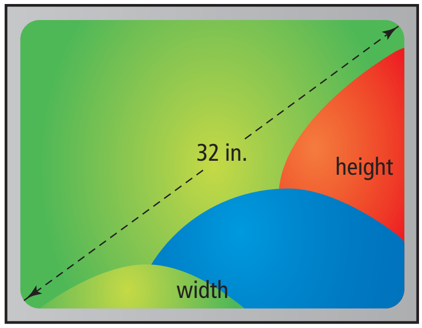
Example 2 Apply Linear Measurement

The Carsons want to buy a 32″ television. The size of a television is measured across the screen diagonally. They are choosing between a standard 4 : 3 television set and a widescreen 16 : 9 HDTV. To help them decide, calculate the screen dimensions and the viewing area for each television. Which television has the greater viewing area?

Solution

Standard 4 : 3 Television Screen

Using the ratio of width to height (4 : 3), draw a scale diagram to visualize the television screen.



Measure the diagonal, the width, and the height of the television screen in the diagram.

diagonal = 2.5 in.
width = 2 in.
height = 1.5 in.

To calculate the width, w, and height, h, of the actual TV screen, calculate the scale factor using the measurement of the diagonal. Let s represent the scale factor.

$$32s = 2.5$$
$$s = \frac{2.5}{32}$$
$$s = 0.078\,125$$

Scale factor(width of actual TV) = width of screen in diagram

$$0.078\,125w = 2$$
$$w = \frac{2}{0.078\,125}$$
$$w = 25.6$$

Scale factor(height of actual TV) = height of screen in diagram

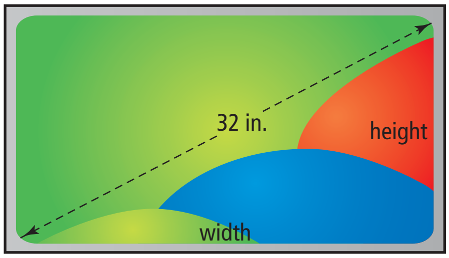
$$0.078\,125h = 1.5$$
$$h = \frac{1.5}{0.078\,125}$$
$$h = 19.2$$

Area of screen = width × height

$$A = 25.6(19.2)$$
$$A = 491.52$$

The viewing area of the standard television is 491.52 in.^{2}.

Widescreen 16 : 9 HDTV



The ratio of width to height in the diagram is 16 : 9.

The actual TV is an enlargement of the diagram.

To determine the dimensions of the actual TV screen, you could use an enlargement factor of x.

Then, the actual width, w, is represented by $16x$ and the actual height, h, is represented by $9x$.

The actual diagonal is 32 in.

You can use the Pythagorean relationship to determine the enlargement factor.

$$\begin{aligned}(16x)^2 + (9x)^2 &= 32^2\\ 256x^2 + 81x^2 &= 1024\\ 337x^2 &= 1024\\ x^2 &= \frac{1024}{337}\\ x^2 &= 3.0385\ldots\\ x &= 1.7431\ldots\end{aligned}$$

Calculate the actual width.

$w = 16x$
$w = 16(1.7431\ldots)$
$w = 27.8904\ldots$

Calculate the actual height.

$h = 9x$
$h = 9(1.7431\ldots)$
$h = 15.6883\ldots$

$$\begin{aligned}\text{Area of screen} &= \text{width} \times \text{height}\\ A &= (27.8904\ldots)(15.6883\ldots)\\ A &= 437.5548\ldots\end{aligned}$$

The viewing area of the standard TV is about 491 in.^{2} and the viewing area of the HDTV is about 438 in.^{2}.

The standard TV has the greater viewing area.

Your Turn

What is the difference in the viewing area for a 46″ standard television (4 : 3) and a 46″ widescreen television (16 : 9)?

Did You Know?

Scientists have developed an ultra-thin flexible screen that folds to fit in a pocket. These screens could be used for computers, telephones, and advertising.

Example 3 Solve a Problem Using Imperial Measurements

Alashun wants to make a drum, or qilaut, that resembles the one used by a drum dancer in Iqaluit, NU. He has a circular frame, over which to stretch caribou skin. Then, he will lash it into place along the frame with sinew. Alashun uses $3\frac{1}{2}$ in. of sinew for each inch of the frame.

Inuit drum dancers performing at the inaugural event in Iqaluit, Nunavut. Nunavut became Canada's newest territory on April 1, 1999.

a) Estimate the diameter of the drum frame in imperial units. The scale of the photo is 1 : 15.

b) Approximately what length of sinew does Alashun need to make the drum? State your answer in yards and inches.

Solution

a) The diameter of the drum in the photo appears to be just over 1″.

$1''(15) \approx 15''$

So, the diameter of the drum is approximately 15″.

How would you estimate the diameter of the drum in the photo? ME

b) Estimate the circumference of the drum frame.

$C = \pi d$

$C = \pi(15)$

$C = 15\pi$

The circumference of the drum frame is approximately 15π inches.

Alashun uses $3\frac{1}{2}$ in. of sinew for each inch of the frame.

Let l represent the length of sinew needed.

$l \approx \left(3\frac{1}{2}\right)(15\pi)$

$l \approx 164.9336...$

Round this distance to whole inches for converting.

Convert 165 in. to yards and inches.

$\frac{165 \text{ in.}}{36} = 4 \text{ yd } 21 \text{ in.}$

What steps do you follow to convert between imperial units?

ME

$\frac{160}{40} = 4$

So, 165 in. $\approx$ 4 yd.

Alashun needs approximately 4 yd 21 in. of sinew.

Your Turn

A round Inuit drum needs to have its skin restretched and then lashed into place with sinew. For each inch of the frame, $3\frac{1}{2}$ in. of sinew are needed. The diameter of the frame is $1\frac{1}{4}$ ft. What length of sinew is needed? Express your answer to the nearest quarter of a foot.

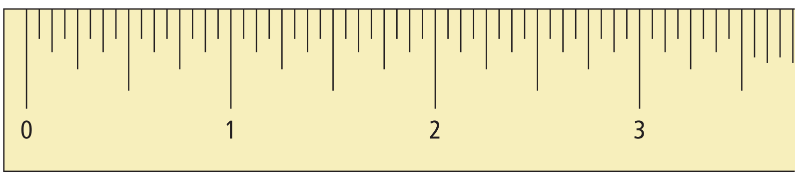
Key Ideas

- The imperial system of measurement is widely used in the United States for measuring distances.
- Even though SI is Canada's official measurement system, some Canadian industries still use imperial units.
- In the imperial system, common units for linear measurement are the inch (in.), foot (ft), yard (yd), and mile (mi). The imperial units for length are related according to the following conversions:
  1 mi = 1760 yd   1 yd = 3 ft   1 ft = 12 in.

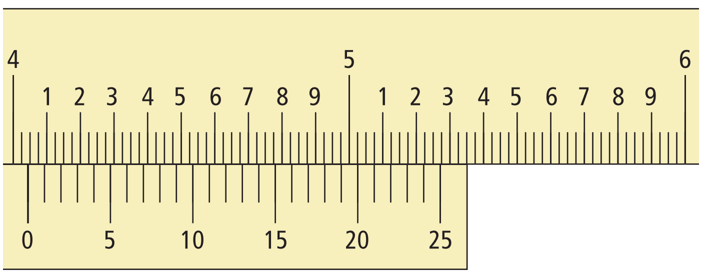
Check Your Understanding

Practise

1. a) What does the smallest subdivision on this imperial ruler represent?



b) Look at the caliper that measures in inches. What is the value of each of the smallest subdivisions on the fixed scale? State your answer as both a fraction and a decimal.



c) What is the value of each of the smallest subdivisions on the moving scale of the caliper in part b)? State your answer in fraction and decimal form.

2. Convert each measurement to the unit indicated.

a) The world's longest earthworm measured 1 ft $1\frac{1}{2}$ in. (nearest half of an inch)

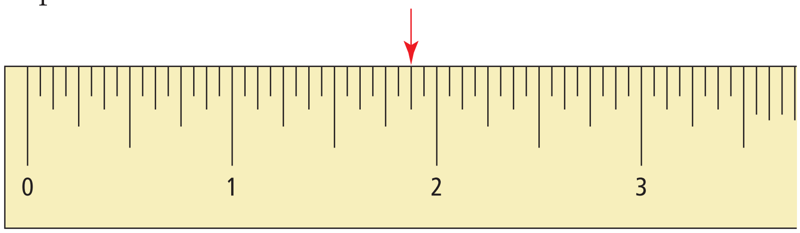
b) The world's shortest man is 2′ 3″. (nearest quarter of a yard)

c) A rocket separates from its space capsule at 400 000 ft. (nearest mile)

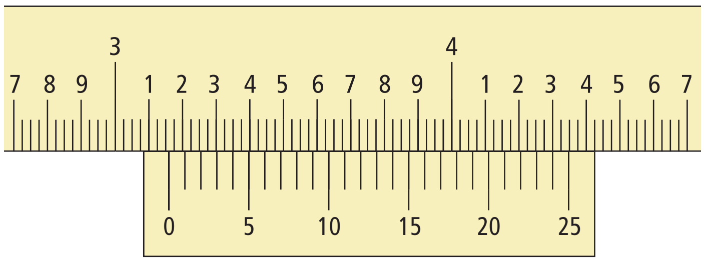
d) The altitude of a balloon is 3 mi. (nearest foot)

3. What reading is shown on each measuring scale? For each measurement, name one item that might have this dimension.

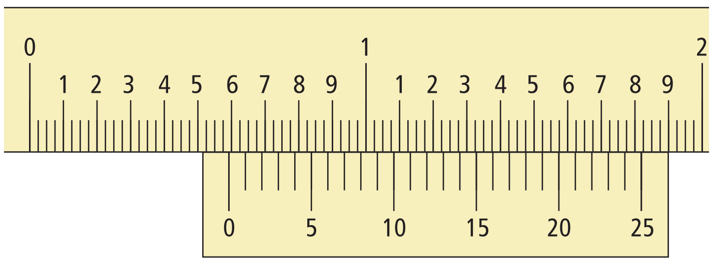
a) imperial ruler



b) imperial caliper



c) imperial caliper



4. Name a measuring device that would be appropriate to measure each distance. Explain your choices. Then, measure each distance, to the nearest sixteenth of an inch.

a) the diameter of a pen

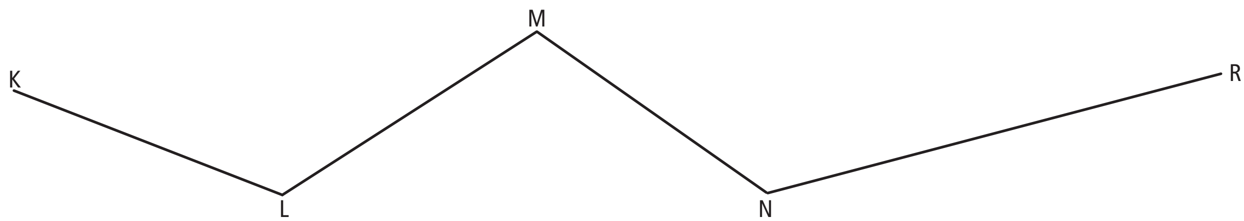
b) the circumference of a pen

c) the length of a pen

5. Use your referent for an inch to estimate the total length of each figure. Then, measure each distance. Express answers to the nearest quarter of an inch.

a)

b)



6. Explain how you can use a personal referent to help you estimate. Then estimate and measure each distance. State each measurement in feet and inches.
   a) the width of your classroom
   b) the perimeter of your desk or table top

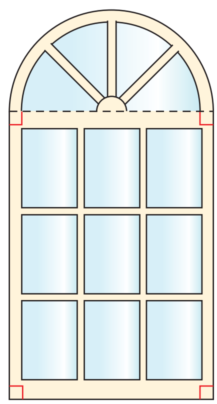
7. Billy Loutit was a Métis mail carrier for the Hudson's Bay Company. He ran 100 mi, from Athabasca, AB, to Edmonton, in 16 h through flooded terrain.
   a) What was Billy's average speed in miles per hour?
   b) How long did it take Billy to run a mile?

Did You Know?

In 1904, William (Billy) Loutit was sent from Athabasca, AB, to Edmonton seeking emergency help against the flood that threatened to destroy Athabasca. By completing the 100-mi journey on foot in 16 h, he became a hero.

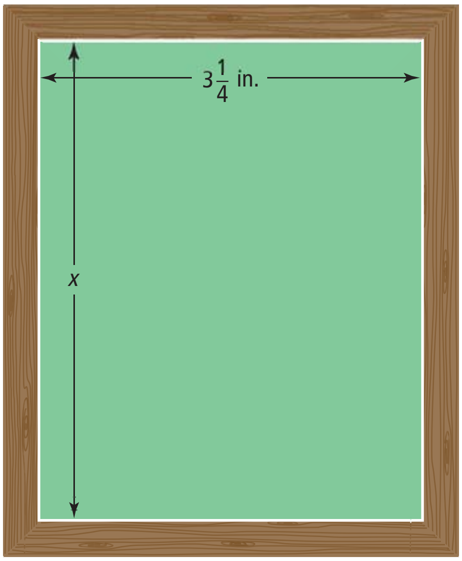
Apply

8. An interior designer wants to present a client with some options for wood trim to frame the Norman window shown in the diagram. The scale of the drawing is 1 : 32. What is the distance around the outside of the window? Express your answer to the nearest half inch. Assume the curve is a semicircle.



9. Leslie has a new manual wheelchair. It has 3 in. diameter micro-caster wheels and 24 in. diameter drive wheels.
   a) Leslie wants to know how many times the caster wheels rotate for each rotation of the drive wheels. Explain the calculations you would perform to obtain the answer. Then, give the answer as a ratio of drive wheel rotations to caster wheel rotations. Write the ratio in lowest terms.
   b) How many rotations of the drive wheels are needed to travel 250 yd?
   c) Suppose Leslie travels $1\frac{1}{2}$ mi. How many rotations will the drive wheels make?

10. Marcus works in a photography laboratory. He needs to enlarge a photograph of Virginia Falls in Nahanni National Park, NT, and make it fit into the frame shown.



   a) What is the scale factor for the enlargement?
   b) What is the length of the unknown side of the frame?

11. Gail and Bram are calculating the area of their washroom floor to order new tiles. Together, they measure the length and width of the floor to be $7\frac{1}{2}$ ft and 5 ft.
   a) Gail calculates the area to be 37 ft^2 5 in.^{2}. Is Gail correct? Explain.
   b) The tiles they select are 6 in. by 12 in. How many tiles are needed to cover their washroom floor?

Did You Know?

The water at Virginia Falls in Nahanni National Park, NT, plunges 295 ft. In the centre of the falls stands Mason's Rock. It is named after Bill Mason, a well-known Canadian adventurer and canoeist.

12. **Unit Project** Today's music storage devices tend to be smaller than those of the past, but they can store many more songs. Find a cassette tape case, a CD, and an MP3 player. Use an imperial unit to measure each of the following dimensions. Justify your choice of unit.

a) the diameter of the CD

b) the dimensions of the cassette case

c) the perimeter of the largest face of the MP3 player

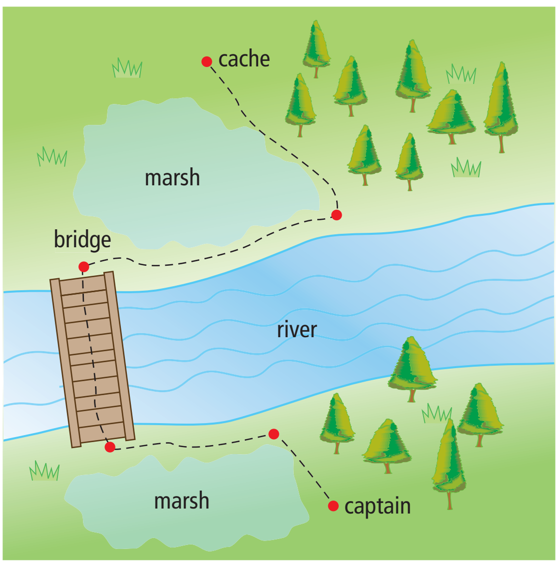
Web Link

To learn more about past and present music storage devices, go to www.mhrmath10.ca and follow the links.

13. A geocaching team captain estimates that a cache is located 500 yd northwest from his position. However, the team must cross the river using the bridge shown.

a) Estimate the distance that the team travels from the captain's position to the cache. Justify your answer.

b) Global Positioning System (GPS) readings provide straight line distances. Estimate the total of the GPS distances between the red dots on the map. Give your answer in yards and feet. How does this total distance compare with the actual distance walked? Explain why.



Extend

14. Sometimes it is difficult to measure the diameter of an object. It may be easier to measure its circumference. The distance around an above-ground circular pool is 60 ft.

a) What is the diameter of the pool? Express your answer to the nearest inch.

b) The owner wants to build a circular wall outside the existing one to help insulate the pool. The material she wants to use is only available in 62 ft, 65 ft, and 70 ft lengths. Determine the diameter of the new insulating wall using each of the available lengths. Express your answer to the nearest inch.

c) Which length would you recommend the owner choose? Explain your reasoning.

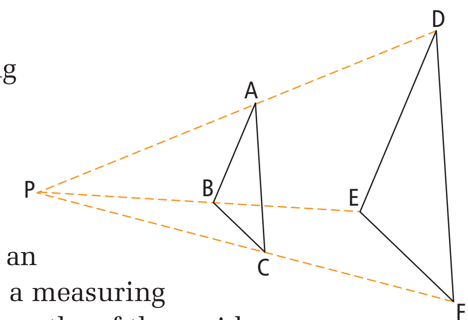
15. The astronomical unit (AU) is a unit of length based on the average distance from Earth to the sun. The AU is currently accepted as 92 955 887.6 mi.

Two comets appeared in rapid succession in 1996 and 1997.
- Comet Hyakutake came within 0.1018 AU of Earth.
- Comet Hale-Bopp came within 1.315 AU of Earth.

a) Within how many miles of Earth did each comet pass?

b) What is the difference in the distance from Earth between the paths of Comets Hale-Bopp and Hyakutake? Give your answer to the nearest mile.

16. You can enlarge a shape by using a point, P, and measuring distances from it to the vertices of the shape and to the enlargement.



a) What ratios of side lengths should be equal if $\triangle DEF$ is an enlargement of $\triangle ABC$? Use a measuring instrument to measure the lengths of these sides and compare the ratios.

b) Explain the mathematics behind this method of enlarging a figure.

c) Use this method to enlarge a figure by a factor of three.

Create Connections

17. Sam's room measures 11 ft by $13\frac{1}{2}$ ft. He wants to buy a new queen-size bed, if it will fit with his existing furniture. Sam has a desk, which measures 4 ft wide by 22 in. deep, and a night stand.

a) Find the dimensions of a double bed and a queen-size bed.

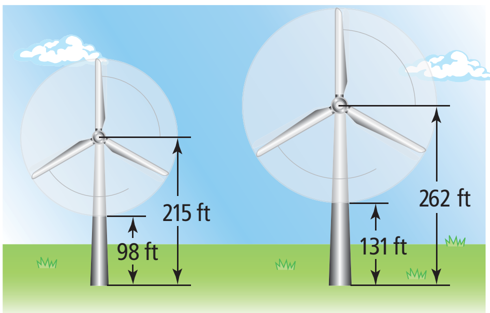
b) Design a layout for Sam's room using a scale diagram. Hint: You will need to estimate the dimensions of a night stand.

c) Which bed do you suggest Sam buy? Why?

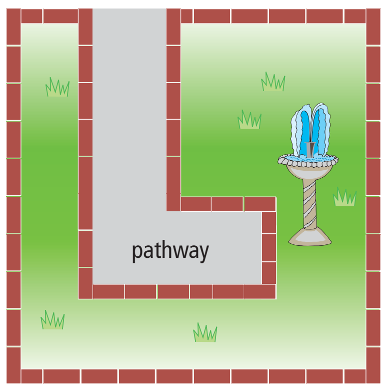
18. Manitoba Hydro announced plans to build a 300-MW wind farm at St. Joseph, MB. Measurements of some of the wind turbines being considered are shown below.

a) What is the difference in the length of the blades for the wind turbines shown?

b) Suppose each turbine makes 30 revolutions per minute. The tip speed of the blades can be up to 6 times the wind speed. Determine the tip speed of each blade. What is the maximum wind speed for each turbine? Hint: speed $= \dfrac{\text{distance}}{\text{time}}$.



19. A pathway leads to a fountain in a small park. The park and the pathway are outlined with a brick border.



a) Estimate the perimeter of the border in the diagram using imperial units. Explain how you estimated your answer.

b) Measure the perimeter. How close was your estimate to the actual measure?

c) Draw a new diagram for the same park but make the pathway to the fountain half the width of the one shown in the diagram. What is the perimeter of the border in your diagram?

d) Predict how the perimeter of the border changes as the width of the pathway changes. Check your prediction. Use words, diagrams, and imperial measurements to support your answer.