

# 9.3

## Solving Problems Using Systems of Linear Equations

### Focus on ...

- choosing a strategy to solve a problem that involves a system of linear equations



### Did You Know?

Dog teams are making a comeback in the Arctic. Part of the reason is that dog teams do not need gas or expensive parts.

Many people drive hybrid vehicles because these cars consume less gas. Some people drive hybrids to reduce their ecological footprint. Others like the savings in the cost of gas. How can you determine if a hybrid is cost efficient?

### Materials

- graphing calculator or computer with spreadsheet software

### Investigate Solving a Problem Involving a System of Linear Equations

A sample price for a hybrid car is \$28 000. The price of a similar car powered by gas is \$21 500. The hybrid vehicle costs \$0.18 per kilometre to operate. The non-hybrid vehicle costs \$0.22 per kilometre to operate.

1. Write a system of linear equations that models the total cost for each vehicle in relation to the distance travelled.

2. Solve the linear system using a graphing calculator or spreadsheet software. After how many kilometres will the hybrid car be more cost efficient?
3. a) Solve the same system of equations from step 1 algebraically. Use either the substitution method or the elimination method.  
b) Explain why you chose the method you did.
4. **Reflect and Respond** Compare and contrast the following three methods of solving a system of linear equations:
  - graphically
  - algebraically by substitution
  - algebraically by elimination
 Include examples.
5. Compare your response to step 4 with a classmate's. Note any insights that your classmate provides.

## Link the Ideas

You can use graphical or algebraic methods to solve systems of linear equations. Each method has its advantages and disadvantages.

Method	Advantages	Disadvantages
Graphical	<ul style="list-style-type: none"> <li>• provides a visual that can show how two variables relate</li> <li>• can be done with or without a graphing calculator</li> <li>• can result in an accurate and quick solution when using a graphing calculator</li> </ul>	<ul style="list-style-type: none"> <li>• can be time-consuming</li> <li>• may not provide an exact solution</li> </ul>
Algebraic	<ul style="list-style-type: none"> <li>• allows for an exact solution relatively quickly</li> <li>• can be done using more than one method (substitution and elimination)</li> </ul>	<ul style="list-style-type: none"> <li>• does not provide any visual insight into how the two variables relate</li> <li>• can result in an incorrect answer due to a minor arithmetic error</li> </ul>

### Example 1 Compare Methods of Solving

Jeremy and Shilan participated in their school's Plant-a-thon fundraiser. Jeremy started planting seedlings at 10:00 a.m. He planted at a steady rate of one tree per minute. Shilan started planting at 11:30 a.m. Her planting rate was three trees every 2 min.



- a) At what time had they planted the same number of trees? Use a system of linear equations to find out. Solve the system using a graphing calculator and algebraically.
- b) Which method do you prefer? Why?

#### Solution

- a) Let  $n$  represent the number of trees that were planted after 11:30 a.m. Let  $m$  represent the number of minutes that have passed since 11:30 a.m. Determine equations for the number of trees planted by Jeremy and Shilan.

For Jeremy:  $n = 90 + 1m$

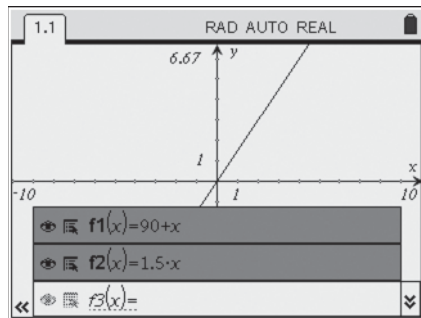
Jeremy started at 10:00 a.m. He planted at a steady rate of one tree per minute. What does 90 represent in the equation?

For Shilan:  $n = 1.5m$

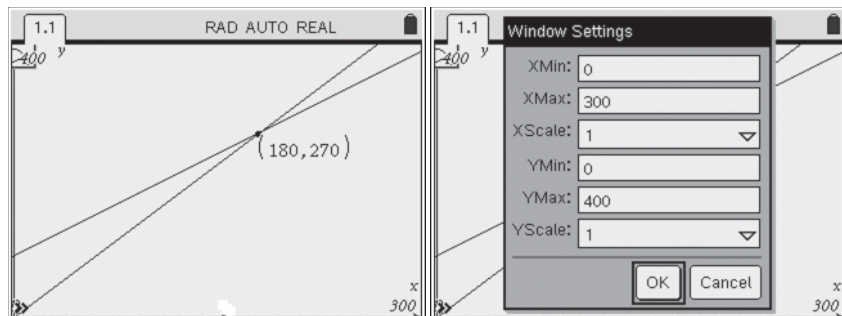
Shilan planted three trees every 2 min. What does 1.5 represent in the equation?

#### Method 1: Solve Using a Graphing Calculator

Graph the equations using a calculator.



Determine an appropriate window and graph it.



The point of intersection is (180, 270).

Check:

Substitute the intersection point into the original equations.

For Jeremy:

$$\begin{array}{rcl} \text{Left Side} & \text{Right Side} & \\ n = 270 & 90 + 1m & \\ & = 90 + 180 & \\ & = 270 & \end{array}$$

Left Side = Right Side

For Shilan:

$$\begin{array}{rcl} \text{Left Side} & \text{Right Side} & \\ n = 270 & 1.5m & \\ & = 1.5(180) & \\ & = 270 & \end{array}$$

Left Side = Right Side

### Method 2: Solve Algebraically

In both equations,  $n$  is isolated. Therefore, the substitution method is an appropriate algebraic method.

Write Shilan's planting rate to be equal to Jeremy's rate.

$$1.5m = 90 + 1m$$

$$0.5m = 90 \quad \text{Solve for } m.$$

$$m = 180$$

Solve for  $n$  by substituting into one of the initial equations.

$$n = 1.5m$$

$$n = 1.5(180)$$

$$n = 270$$

Check:

Substitute into the original equations.

For Jeremy:

$$\begin{array}{rcl} \text{Left Side} & \text{Right Side} & \\ n = 270 & 90 + 1m & \\ & = 90 + 180 & \\ & = 270 & \end{array}$$

Left Side = Right Side

For Shilan:

$$\begin{array}{rcl} \text{Left Side} & \text{Right Side} & \\ n = 270 & 1.5m & \\ & = 1.5(180) & \\ & = 270 & \end{array}$$

Left Side = Right Side

Both Jeremy and Shilan had planted 270 trees 180 min after Shilan started. 180 min = 3 h and 11:30 a.m. + 3 h = 2:30 p.m. They had planted the same number of trees at 2:30 p.m.

- b) The algebraic method is preferable. For the graphing method, the equations were already in the form  $y = mx + b$ , so it was easy to graph. However, it took some time to find a reasonable window to determine the intersection point. For the algebraic method, the substitution method was easy to use, since both equations were in the form  $y = mx + b$ .

### Your Turn

Solve the following system of linear equations algebraically and graphically. Which method do you prefer? Explain.

$$y = 0.25x - 200$$

$$3x + 2y = 160$$

### Did You Know?

There are numerous multicultural festivals across western Canada. People celebrate and learn about the culture, food, and entertainment of various ethnic and indigenous groups. Every August, the Folklorama Festival in Winnipeg and Folkfest in Saskatoon have a Métis pavilion.

## Example 2 Compare Algebraic Methods



Folkfest, Saskatoon

At the Métis People Pavilion, visitors can enjoy bannock and buffalo stew. A recent sale of three orders of stew and two orders of bannock cost \$13.50. A second sale of four orders of stew and five orders of bannock cost \$21.50.

- a) Use a system of linear equations to determine the price of one order of bannock and the price of one order of stew. Solve the system algebraically using two methods.
- b) Compare the two methods.

### Solution

- a) Let  $B$  represent the cost for one order of bannock, in dollars. Let  $S$  represent the cost for one order of buffalo stew, in dollars. Write an equation to represent the first sale.

$$3S + 2B = 13.50 \quad \textcircled{1}$$

Write an equation to represent the second sale.

$$4S + 5B = 21.50 \quad \textcircled{2}$$

#### Method 1: Use Substitution

Isolate the variable  $B$  in  $\textcircled{1}$  since it has the smallest coefficient, 2.

$$\begin{aligned} 3S + 2B &= 13.50 \\ 2B &= 13.50 - 3S \\ B &= 6.75 - 1.5S \end{aligned}$$

You can try isolating the variable  $S$  in  $\textcircled{1}$  instead.

$$\begin{aligned} 3S + 2B &= 13.50 \\ 3S &= 13.50 - 2B \\ S &= 4.5 - \frac{2}{3}B \end{aligned}$$

Why might it be better to isolate  $B$ ?

Substitute the expression for  $B$  into  $\textcircled{2}$ .

$$\begin{aligned} 4S + 5(6.75 - 1.5S) &= 21.50 \\ 4S + 33.75 - 7.5S &= 21.50 \\ -3.5S + 33.75 &= 21.50 \\ -3.5S &= -12.25 \\ S &= 3.50 \end{aligned}$$

What does 3.50 represent?

Substitute the value of  $S$  into  $\textcircled{1}$  or  $\textcircled{2}$  to solve for  $B$ .

$$\begin{aligned} 3(3.50) + 2B &= 13.50 \\ 10.50 + 2B &= 13.50 \\ 2B &= 3.00 \\ B &= 1.50 \end{aligned}$$

What does 1.50 represent?

**Method 2: Use Elimination**

Multiply ① by 5. Multiply ② by  $-2$ . Then, eliminate the variable  $B$  by addition.

$$\begin{array}{rcl} 5(3S + 2B) = 5(13.50) & & -2(4S + 5B) = -2(21.50) \\ 15S + 10B = 67.50 & \textcircled{3} & -8S - 10B = -43.00 & \textcircled{4} \end{array}$$

Add ③ and ④ to eliminate the variable  $B$ .

$$\begin{array}{rcl} 15S + 10B & = & 67.50 \\ + (-8S - 10B) & = & -43.00 \\ \hline 7S & = & 24.50 & \text{Solve for } S. \\ S & = & 3.50 \end{array}$$

Substitute the value of  $S$  into ① or ② to solve for  $B$ .

$$\begin{aligned} 3(3.50) + 2B &= 13.50 \\ 10.50 + 2B &= 13.50 \\ 2B &= 3.00 \\ B &= 1.50 \end{aligned}$$

Check:

Substitute into ① and ②.

Left Side	Right Side
$3S + 2B$	13.50
$= 3(3.50) + 2(1.50)$	
$= 10.50 + 3.00$	
$= 13.50$	

Left Side = Right Side

Left Side	Right Side
$4S + 5B$	21.50
$= 4(3.50) + 5(1.50)$	
$= 14.00 + 7.50$	
$= 21.50$	

Left Side = Right Side

The price of one order of stew is \$3.50. The price of one order of bannock is \$1.50.

- b)** For this question, it took more steps to isolate the first variable using the substitution method than the elimination method. Also, if the variable  $S$  had been isolated in ①, the result for  $S$  would have made it complicated to solve for  $B$ .

**Your Turn**

Solve the linear system twice, using both algebraic methods. Compare the two methods.

$$\begin{aligned} 3x - 4y &= 17 \\ 4x + 5y &= 48.5 \end{aligned}$$

## Key Ideas

- Systems of linear equations can be solved
  - graphically
  - algebraically by substitution or by elimination
- It may be better to use a graphical approach to solve linear equations when you wish to see how the two variables relate, such as for cost analysis and speed problems.
- It may be better to use an algebraic approach to solve linear equations when
  - you need only the solution (intersection point)
  - it is unclear where to locate the solution on a coordinate plane

## Check Your Understanding

### Practise

1. Solve each system of linear equations using a method of your choice. Check your answer graphically.

a)  $2x - 5y = 12$   
 $-7x + 5y = 48$

b)  $3y = 6 - x$   
 $5x + 6y = -6$

c)  $n = 3k - 2$   
 $2n - 6k = -4$

2. Solve each system of linear equations using your preferred method.

a)  $0.2y + x = 0.7$   
 $2y + 12x = 11$

b)  $\frac{m}{7} + \frac{n}{2} = 7$   
 $2m + 6 = 3n$

c)  $4x - 7y = 6$   
 $5x = 2y + 3$

### Apply

3. In January, the average high temperature for Calgary is  $9.9^\circ\text{C}$  greater than Winnipeg's average high temperature. The sum of these two temperatures is  $-15.5^\circ\text{C}$ . What is the average high temperature in January for each of these two cities?



4. In Canada, the percent of workers who drive themselves to work is approximately 11.3 times the percent of workers who walk. The combined percent of Canadian workers who drive themselves and walk is about 78.7%. Approximately what percent of Canadian workers walk to work? Express your answer to the nearest tenth of a percent.
5. A school's multicultural club is selling muffins for a fundraiser. The club spends \$16.00 on advertising. The cost of ingredients for each muffin is \$0.30. The club decides to sell the muffins for \$0.75 each. The following equations model this situation:  $C = 0.3m + 16$  and  $C = 0.75m$ .
- Describe in words what each equation represents.
  - Determine the minimum number of muffins the club will have to sell to cover their total costs.
6. An incandescent 60-W light bulb sells for approximately \$0.75. It costs \$0.0072 to operate per hour. An equivalent compact fluorescent bulb costs \$4.00. It uses 15 W of power and costs \$0.0018 per hour to operate. The following equations model the cost:  $C = 0.75 + 0.0072h$  and  $C = 4 + 0.0018h$ . In these equations,  $C$  is the total cost, in dollars, and  $h$  is the number of hours. How many hours will it take for the compact fluorescent bulb to be less expensive?

#### Did You Know?

A compact fluorescent bulb has a lifespan of approximately 10 000 h versus 1000 h for an incandescent bulb.

7. A circus recently had a sold-out performance. There were varying admission prices. The admission for premium seating was \$250 for adults and \$175 for students. The total revenue for premium seating was \$29 125. The receipts showed that 130 premium seats were sold. Determine how many adults and how many students were in premium seats.



Cirque du Soleil

#### Did You Know?

Cirque du Soleil is a dramatic mix of circus arts and street entertainment. Based in Montréal, Québec, it was founded in 1984. The founders were two former street performers, Guy Laliberté and Daniel Gauthier. Cirque expanded rapidly. It grew from one show to multiple shows around the world.

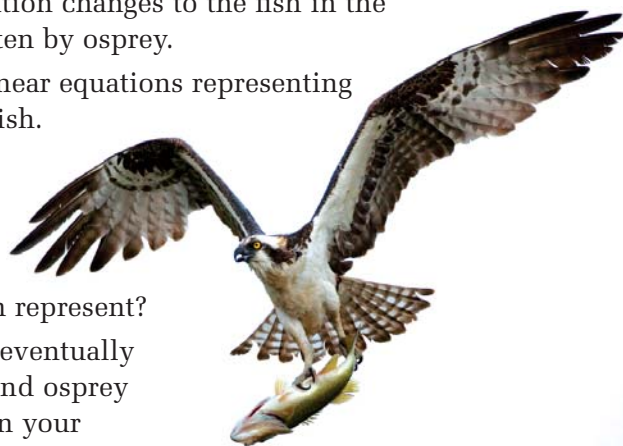
8. Jason is renting a car for one week. Speed-E-Car Rental offers a compact car for \$379 plus \$0.10 per kilometre. Easy 4 U Auto offers a compact car for \$249 plus \$0.35 per kilometre. Use a system of linear equations to determine when each company would be the better choice for Jason.



9. **(Unit Project)** The water level in a lake is decreasing. Wildlife biologists are concerned about the effect on the fish population. They decide to track the number of fish in the lake. The osprey is a fish-hunting bird. As part of their study, the biologists need to estimate the number of fish eaten by osprey.

Year	Fish in Lake	Fish Eaten by Osprey
1	10 000	700
2	9 000	900
3	8 000	1100
4	7 000	1300

- Describe the population changes to the fish in the lake and the fish eaten by osprey.
- Write a system of linear equations representing the populations of fish.
- Solve the system of linear equations graphically. What does the point of intersection represent?
- Predict what might eventually happen to the fish and osprey populations. Explain your thinking.



### Did You Know?

Scuba divers can suffer from a potentially lethal condition called *decompression sickness*, or *the bends*. This occurs if they rise too quickly to the surface of the water. At greater depths, there is extra pressure on the diver. The extra pressure causes nitrogen gas to dissolve in the diver's blood. As the diver slowly rises back to the surface, the nitrogen gas comes out of solution and forms bubbles in the blood. If the diver ascends too quickly, the nitrogen remains dissolved in the blood. The result is the bends, a painful condition that may be fatal.

10. Scuba divers can spend only a limited amount of time at depths between 60 m and 90 m. This amount of time can be represented by a linear relation. A diver can remain for 60 min at a depth of 60 m, and 30 min at a depth of 90 m. Write a system of linear equations to determine the slope intercept form,  $y = mx + b$ , for this linear relation.



11. Last Saturday, Juan went cross-country skiing in the morning. In the afternoon, he played squash. Cross-country skiers expend 50 kJ of energy per minute. Squash players burn 42 kJ per minute. In total, Juan exercised for 100 min. He used 4850 kJ in energy. How much time did he spend doing each activity?



### Extend

12. The lines that enclose a triangle can be represented by graphs of the equations  $y = 3$ ,  $y = -x + 7$ , and  $y = 2x + 16$ . Use a system of linear equations to determine the area of the triangle.
13. Answer the following questions using the two systems of linear equations shown.
- $$\begin{array}{l} 39x + 49y = 2283 \\ 43x + 54y = 2516 \end{array} \quad \text{and} \quad \begin{array}{l} 39x + 49y = 2283 \\ 43x + 54y = 2517 \end{array}$$
- a) What difference do you see between the two systems of linear equations just by looking at them?
- b) Solve both systems algebraically.
- c) Solve both systems graphically.
- d) Explain why these linear systems are difficult to solve.

### Create Connections

14. a) Create a system of linear equations. Solve your system using a method of your choice.
- b) How do you decide on a strategy for solving a system of linear equations? What do you consider? Why?
15. a) Create a system of linear equations with a solution involving a rational number that cannot be expressed exactly on a graphing calculator. Solve your system graphically and algebraically.
- b) Describe your results using the two methods.