

7.2

General Form

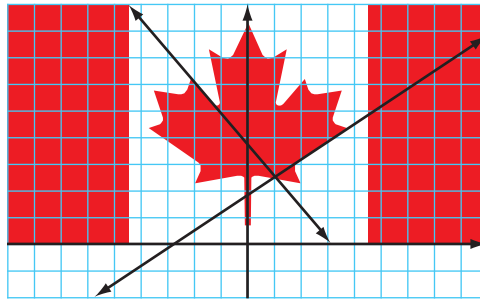
Focus on ...

- converting a linear equation to general form
- using intercepts to graph a line
- relating the intercepts of a graph to the situation
- solving problems using equations in general form

Materials

- bottle of water
- stopwatch
- grid paper and ruler or graphing technology

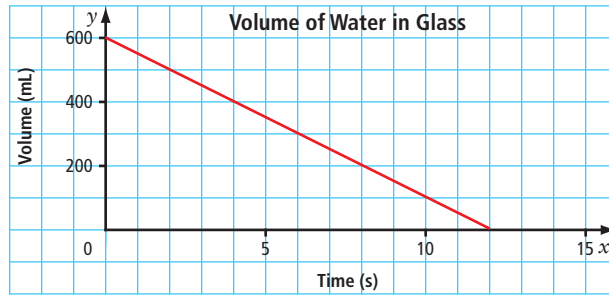
The slope-intercept form of an equation is one of the most common ways to write the equation of a line. Suppose you place a transparent grid on the Canadian flag. Most, but not all, of the line segments in the flag could be expressed as equations in slope-intercept form. Can you think of a type of line that cannot be expressed in the form $y = mx + b$? In this section, you will explore a general way of writing the equation of any line.



Investigate Intercepts and General Form

Leora quenches her thirst after a soccer game by drinking a large glass of water at a constant rate. The straight-line graph on the following page shows how the volume of water in the glass changes with time.





1. Identify the domain and range in this situation.
2. Identify the slope of the line segment. What does the sign of the slope mean? What does the slope represent?
3. What is the y-intercept of the line segment? What does the y-intercept represent?
4. a) What is the equation of the line in slope-intercept form?
b) Rearrange the terms in the equation so that the right side is zero. This is the **general form** of an equation.
5. a) Identify the **x-intercept** of the line. What does the x-intercept represent?
b) Write the coordinates of the x-intercept and the y-intercept.
6. Time yourself while drinking a 500-mL bottle of water slowly at a constant rate.
 - a) Sketch a straight-line graph to show how the volume of water remaining in the bottle changes with time.
 - b) Compare your graph to Leora's graph. How can you tell who drank more, who finished first, and who drank at a faster rate?
 - c) Write the equation of your graph in both slope-intercept form and general form.
7. **Reflect and Respond**
 - a) Recall that a point on a line satisfies the equation of the line. Are the x-intercept and y-intercept points on the line?
 - b) When asked to determine the x-intercept, what coordinate value do you always know? When asked to determine the y-intercept, what coordinate value do you always know? How can you use this information to help you determine the x-intercept or y-intercept?
 - c) Describe strategies for determining the y-intercept and x-intercept of a line if the equation is given in general form.
 - d) Create two linear equations. Then, identify the y-intercept and x-intercept of each line. Use graphing technology to check your answers.
8. What form is the equation $x - 5 = 0$ expressed in? Can you write the equation in another form? Explain.

general form

- the equation of a line in the form $Ax + By + C = 0$, where A , B , and C are real numbers, and A and B are not both zero. By convention, A is a whole number. This means that A will always be positive.

x-intercept

- the x-coordinate of the point where a line or curve crosses the x-axis
- the value of x when $y = 0$

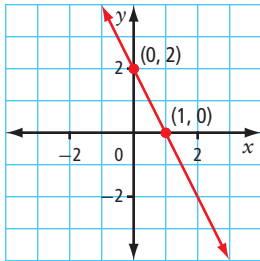
Link the Ideas

The general form of a linear equation is $Ax + By + C = 0$, where A , B , and C are real numbers, and A and B are not both zero. By convention, A is a whole number.

You can convert a linear equation from one form to another by applying the rules of algebra.

The x -intercept of a line is the x -coordinate of the point where the line crosses the x -axis. The y -intercept is the y -coordinate of the point where the line crosses the y -axis. To sketch a linear equation, you can draw a line joining the intercepts, $(x, 0)$ and $(0, y)$.

The line in the graph below has an x -intercept of 1 and a y -intercept of 2.



Example 1 Convert an Equation to General Form

Rewrite the equation $y = -\frac{2}{3}x + 6$ in general form, $Ax + By + C = 0$.

Solution

$$y = -\frac{2}{3}x + 6$$

$$3(y) = 3\left(-\frac{2}{3}x + 6\right) \quad \text{Why are both sides multiplied by 3?}$$

$$3y = \overset{1}{\cancel{3}}\left(-\frac{\overset{1}{\cancel{3}}2}{\overset{1}{\cancel{3}}3}x\right) + 3(6)$$

$$3y = -2x + 18$$

$$2x + 3y - 18 = 0$$

The equation written in general form is $2x + 3y - 18 = 0$.

Your Turn

Rewrite the equation $y = \frac{3}{4}x - 2$ in general form.

Example 2 Sketch a Graph Using Intercepts

For the linear equation $2x - 3y - 6 = 0$,

- state the x -intercept of a graph of the equation
- state the y -intercept
- use the intercepts to graph the line

Solution

- a) To determine the x -intercept, substitute $y = 0$. Then, solve for x .

$$2x - 3y - 6 = 0$$

$$2x - 3(0) - 6 = 0$$

$$2x - 6 = 0$$

$$2x = 6$$

$$x = 3$$

Why do you substitute $x = 0$ to find the y -intercept?

The x -intercept is 3. So, the line crosses the x -axis at the point $(3, 0)$.

- b) To determine the y -intercept, substitute $x = 0$. Solve for y .

$$2x - 3y - 6 = 0$$

$$2(0) - 3y - 6 = 0$$

$$-3y - 6 = 0$$

$$-6 = 0 + 3y$$

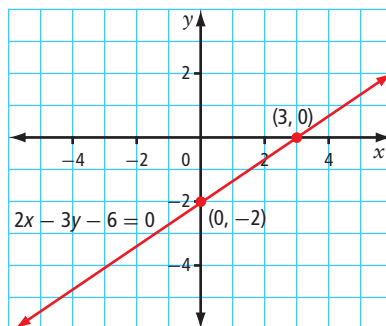
$$\frac{-6}{3} = y$$

$$-2 = y$$

Why do you substitute $y = 0$ to find the x -intercept?

The y -intercept is -2 . The line crosses the y -axis at the point $(0, -2)$.

- c) Locate the points $(3, 0)$ and $(0, -2)$ on the grid. Then, draw a line passing through these points.



How else could you graph an equation given in general form?

Your Turn

Consider the linear equation $4x + 5y - 20 = 0$.

- What is the x -intercept of a graph of the equation?
- What is the y -intercept?
- Use the intercepts to graph the line.

Example 3 Identify Intercepts of Horizontal or Vertical Lines

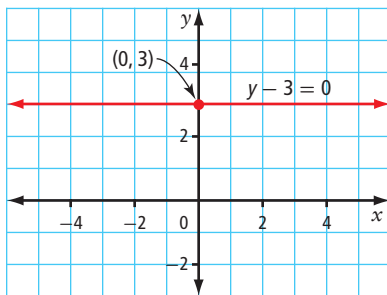
Sketch each linear relation and identify the intercepts.
Then, state the domain and range.

- a) $y - 3 = 0$
- b) $x + 4.5 = 0$
- c) $y = 0$

Solution

- a) The equation $y - 3 = 0$ can be written in slope-intercept form as $y = 0x + 3$.

The graph is a horizontal line with slope zero.
The line crosses the y -axis at the point $(0, 3)$.



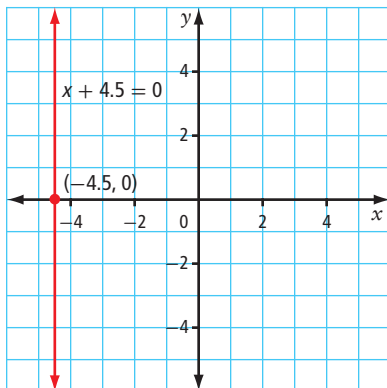
What would the values in a table representing this equation show?

The y -intercept is 3. There is no x -intercept.
The domain of the line $y - 3 = 0$ is $\{x \in \mathbb{R}\}$.
The range of the line is $\{3\}$.

- b) The equation $x + 4.5 = 0$ expressed in general form is $x + 0y + 4.5 = 0$. The coefficient of y is zero.

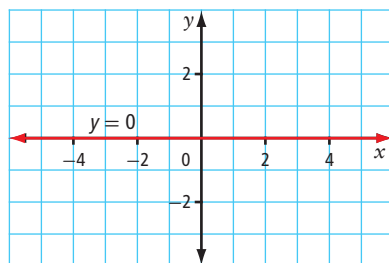
The value of x is always -4.5 .

The graph is a vertical line and crosses the x -axis at $(-4.5, 0)$.



The x -intercept is -4.5 . There is no y -intercept.
The domain of the line $x + 4.5 = 0$ is $\{-4.5\}$.
The range of the line is $\{y \in \mathbb{R}\}$.

- c) The equation $y = 0$ is a horizontal line, which represents the entire x -axis. The graph always intersects the x -axis. Therefore, there are an infinite number of x -intercepts. The y -intercept is 0. The domain of the line $y = 0$ is $\{x \in \mathbb{R}\}$. The range of the line is $\{0\}$.



What do the coordinates of points on this line have in common?

Your Turn

Sketch each linear relation and identify the intercepts. What are the domain and range for each relation?

- a) $x - 3 = 0$
- b) $x = 0$
- c) $y + 2 = 0$

Example 4 Interpret Intercepts

Spencer has 66 GB of disk space left on his laptop to fill with television shows and movies that he purchases on-line.



- a) Suppose a one-hour show uses 1.1 GB of disk space and a movie uses 4.4 GB. Write a linear equation that represents the number of television shows, T , and movies, M , that Spencer can store on his laptop.
- b) Determine the T -intercept of a graph of the linear equation. What does the T -intercept represent?
- c) What would the M -intercept be? What does the M -intercept represent?
- d) If Spencer stores 16 television shows, how many movies does he have space for?

Solution

- a) The equation is $1.1T + 4.4M = 66$.

What does each term represent?

Simplify the equation.

Write the coefficient of T as a whole number.

$$11T + 44M = 660$$

Reduce the equation to lowest terms.

$$T + 4M = 60$$

How else could you reduce the original equation to lowest terms with the coefficient of T as a whole number?

The equation $T + 4M = 60$ represents the number of television shows and movies that Spencer can store on his laptop.

- b) To determine the T -intercept, substitute $M = 0$. Solve for T .

$$T + 4M = 60$$

$$T + 4(0) = 60$$

$$T = 60$$

The T -intercept is 60. So, if Spencer stores no movies, he can store 60 television shows.

- c) To determine the M -intercept, substitute $T = 0$. Solve for M .

$$T + 4M = 60$$

$$0 + 4M = 60$$

$$M = 15$$

The M -intercept is 15. So, Spencer can store 15 movies if he does not store any television shows.

- d) Substitute $T = 16$ and solve for M .

$$T + 4M = 60$$

$$16 + 4M = 60$$

$$4M = 44$$

$$M = 11$$

Spencer has space on his laptop for 11 movies.

Your Turn

Brooke wants to save \$336 to decorate her bedroom. She has two part-time jobs. On weekends, she works as a snowboard instructor and earns \$12 per hour. On weeknights, she earns \$16 per hour working as a high-school tutor.

- a) Write an equation to represent the number of hours Brooke needs to work as a snowboard instructor, S , and as a tutor, T .
- b) What is the S -intercept of a graph of the equation? What does the S -intercept represent?
- c) What would the T -intercept be? What does it represent?
- d) Suppose Brooke works 8 h as a snowboard instructor. How many hours will she need to work as a tutor?



Key Ideas

- The general form of a linear equation is $Ax + By + C = 0$, where A , B , and C are real numbers, and A and B are not both zero. By convention, A is a whole number.
- To graph an equation in general form, determine the intercepts, then draw a line joining the intercepts; or convert to slope-intercept form.
- To determine the x -intercept, substitute $y = 0$ and solve. To determine the y -intercept, substitute $x = 0$ and solve.
- A sketch of a linear relation may have one, two, or an infinite number of intercepts. A line that represents an axis has an infinite number of intercepts with that axis. A horizontal or vertical line that does not represent an axis has only one intercept.

Equation	x -Intercept(s)	y -Intercept(s)	Graph
$x + 2y - 3 = 0$	$x + 2y - 3 = 0$ $x + 2(0) - 3 = 0$ $x = 3$	$x + 2y - 3 = 0$ $(0) + 2y - 3 = 0$ $2y = 3$ $y = 1.5$	
$x = 5.3$	$x = 5.3$	no y -intercept	
$3y = 0$	infinite number of x -intercepts	$3y = 0$ $y = 0$	

Check Your Understanding

Practise

1. Jasmine was asked to convert the equation $y = -\frac{3}{2}x + 4$ to general form. Her work is shown.

$$\begin{aligned}y &= -\frac{3}{2}x + 4 \\2y &= -3x + 4 \\3x + 2y - 4 &= 0\end{aligned}$$

Identify Jasmine's error. Then, correct her work.

2. Express each equation in general form, $Ax + By + C = 0$.

a) $y = 7x - 5$

b) $y = -x + 8$

c) $y = \frac{3}{2}x + 4$

d) $y = -\frac{3}{5}x - 2$

e) $y = 0.25x - 0.3$

f) $y = -\frac{5}{2}x + \frac{1}{8}$

3. Determine the intercepts of each line. Then, graph the line.

a) $2x + y - 9 = 0$

b) $4x - y - 8 = 0$

c) $x - 2y + 10 = 0$

d) $3x - 8y - 24 = 0$

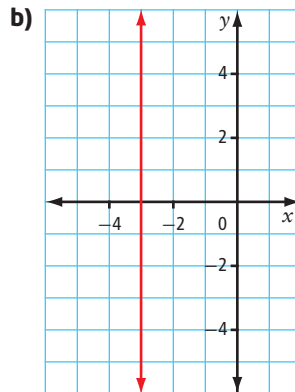
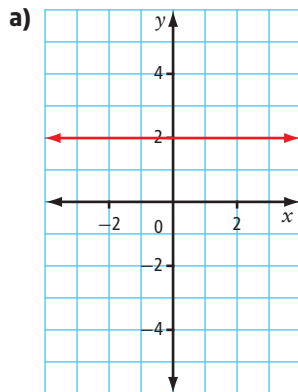
e) $4x + 5y + 6 = 0$

f) $x = 4$

g) $y = 0$

h) $4x - 12 = 0$

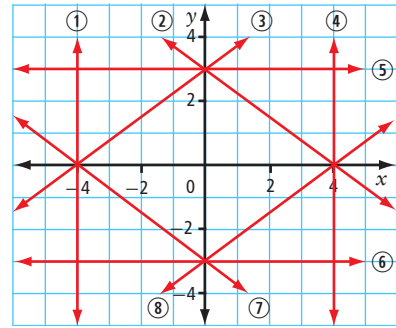
4. For each line, state the domain, range, intercepts, and slope. What is the equation of each line, in general form?



5. Graph each line using the given intercepts. What is the equation of each line?
- an x-intercept of 3 and no y-intercept
 - a y-intercept of -5 and no x-intercept
 - an infinite number of x-intercepts
 - an infinite number of y-intercepts

6. Match each equation with a line labelled in the figure.

- $3x + 4y = 12$
- $x = 4$
- $3x - 4y = 12$
- $y - 3 = 0$
- $3x - 4y + 12 = 0$
- $y = -3$
- $3x + 4y + 12 = 0$
- $x + 4 = 0$

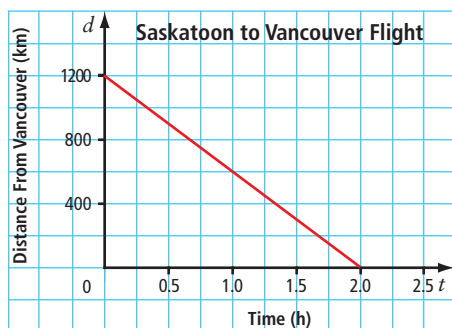


Apply

7. Write an equation, in general form, for each line described.
- a vertical line passing through the point $(3, 5)$
 - a horizontal line passing through the point $(-2, 6)$
 - the x-axis
 - the y-axis
8. Write an equation, in general form, of a line that does not have a y-intercept and passes through the point $(3, 6)$.
9. Having fibre in your diet helps with digestion, heart health, and maintaining a healthy weight. Courtney wants to increase her fibre intake by 21 g per day. She plans to do this by adding bran buds and one vegetable to her diet each day. Write an equation in general form that describes this situation. Courtney mixes 125 mL of bran buds in her cereal each day. What volume of green peas would she need to eat during the day? What volume of baked beans would she need to eat? Express your answers in whole millilitres.

Food	Grams of Fibre per 125 mL
Green peas	4
Baked beans	7
Bran buds	16

10. An airplane flies directly from Saskatoon, SK, to Vancouver, BC. The graph shows the relationship between the distance from Vancouver, d , in kilometres, and the flying time, t , in hours.



- State the intercepts of the line segment. What does each intercept represent?
 - State a suitable domain and range of the graph.
 - Determine the slope of the line. What does the slope represent?
 - Write the equation of the line in general form.
 - For how many hours has the plane been flying when it is 200 km from Vancouver?
 - What is the distance from Vancouver when the plane has been flying for 45 min?
11. Luc swims as part of an active and healthy lifestyle. The number of calories burned by a swimmer of Luc's body weight is shown in the table.

Swimming Style	Calories Burned Per Minute
Backstroke	8
Butterfly	11

- Write a linear equation to describe the number of minutes Luc would need to swim backstroke, x , and butterfly, y , to burn 440 cal.
- What are the intercepts of the line? What does each intercept represent?
- Suppose Luc swims butterfly for 16 min. How long will he need to swim backstroke in order to burn 440 cal in total?





12. Sanding trucks spread a mixture of sand and salt on roads to improve traction in winter. The density of the salt is 1200 kg/m^3 . The density of the sand is 1800 kg/m^3 .

- Write a linear equation to represent the volume, in cubic metres, of salt, x , and of sand, y , in a mixture with a mass of $10\,000 \text{ kg}$.
- For temperatures below -12°C , the volume of sand in $10\,000 \text{ kg}$ of the mixture is 5.22 m^3 . What is the volume of salt in the mixture?
- By mass, what percent of the mixture is salt?

13. Advance tickets for a local concert sold for $\$8$ each. Tickets at the door were $\$12$ each. The revenue from ticket sales was $\$1120$.

- Write a linear equation relating the number of advance tickets, a , to the number of tickets sold at the door, d . Express your equation in general form.
- Describe the steps you would follow to graph this equation. Do not graph.
- Suppose twice as many advance tickets were sold as tickets at the door. How many of each type of ticket were sold?



14. What is the value of the unknown parameter in each equation?

- $Ax + 5y - 6 = 0$, passing through $(-3, 2)$
- $2x + By + 7 = 0$, passing through $(4, -5)$
- $4x - 3y + C = 0$, passing through $(-2, -6)$

Extend

15. The equation of a line is $x + 3y - 24 = 0$. Write the coordinates of a point on the line for each of the following conditions.

- The x -coordinate is equal to the y -coordinate.
- The x -coordinate is three times as great as the y -coordinate.
- The y -coordinate is four greater than the x -coordinate.

16. The equation $6x + By + 5 = 0$ describes a line with a slope of $\frac{3}{2}$. What is the value of B ?

17. What is the area of the triangle bounded by each set of lines?
- the line $x + 2y = 10$, a line with an infinite number of x -intercepts, and a line with an infinite number of y -intercepts
 - a line with an infinite number of x -intercepts, the line $2x - y = 6$, and the line $x = 10$

Create Connections

18. a) Which form of a linear equation do you prefer to graph: $y = mx + b$ or $Ax + By + C = 0$? Why?
- b) Describe a situation when you might work with a linear equation in the other form.
19. The equation of a line is $2x + y - 8 = 0$.
- Explain how you could determine the x -intercept.
 - Explain two different ways to determine the y -intercept.
20. The general form of an equation is $Ax + By + C = 0$. Identify each line as horizontal, vertical, or oblique.
- $A \neq 0, B = 0$, and C is a real number.
 - $A = 0, B \neq 0$, and C is a real number.
 - $A \neq 0, B \neq 0$, and C is a real number.

21. **MINI LAB** Explore the effects of changing parameters on a graph of $Ax + By + C = 0$.

Step 1 For each group of equations, graph the three linear relations on the same axes. Use graphing technology or sketch the graphs by hand.

- | | |
|---------------------|---------------------|
| a) $x + y - 2 = 0$ | b) $x + 2y - 6 = 0$ |
| $x + y - 6 = 0$ | $2x + 2y - 6 = 0$ |
| $x + y - 8 = 0$ | $3x + 2y - 6 = 0$ |
| c) $2x + y - 6 = 0$ | d) $x + y - 2 = 0$ |
| $2x + 2y - 6 = 0$ | $2x + 2y - 4 = 0$ |
| $2x + 3y - 6 = 0$ | $3x + 3y - 6 = 0$ |

Step 2 Compare the three graphs in each group of equations. What parameter(s) have changed? Explain how changing one or two parameters affects the slope, x -intercept, and y -intercept of the graph.

Step 3 How does changing the parameters A , B , and C affect the graph of a linear equation in general form?

Materials

- graphing technology or grid paper and ruler