6.4 Functions

Focus on ...

- sorting relations into functions and non-functions
- using notation specifically designed for functions
- graphing linear functions

There is a special class of relations, called functions, where two quantities depend on each other in a particular way. The amount of tension on a guitar string determines the musical note played. The channel displayed on your television screen depends on the number you enter into the remote. Describe other examples from your daily life where two quantities depend on each other in a particular way.

Investigate Functions

Study the following relations. They are categorized as functions and non-functions.

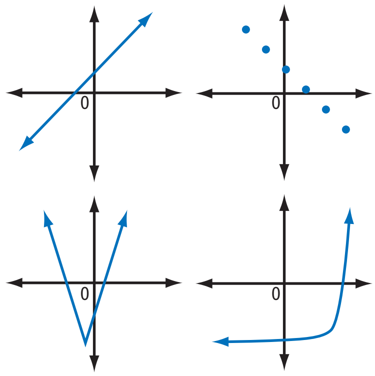
These 8 relations are functions.

x	*y*
5	10
6	15
7	20

x	*y*
11	3
21	3
31	3

{(−2, −5), (0, 4), (2, 13), (4, 22)}

{(10, 10), (12, 10), (14, 12), (16, 12)}



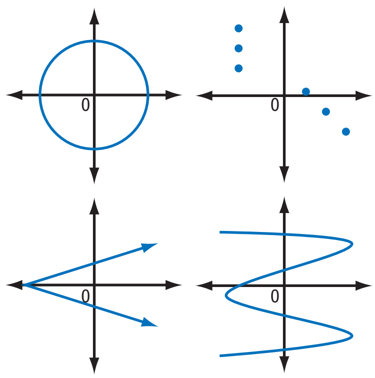
These 8 relations are not functions.

x	*y*
6	10
6	15
7	20

x	*y*
3	11
3	21
3	31

{(10, 10), (12, 10), (12, 14), (12, 16)}

{(7, 5), (7, 8), (9, 11), (11, 14)}



1. What is similar about the functions? What is similar about the non-functions? Discuss your findings with a partner.

2. Reflect and Respond

a) How can you tell whether or not a relation is a function?.

b) Share your explanation with your classmates. Once you have heard other explanations, revise yours to be more precise.

Link the Ideas

function

- a relation in which each value of the independent variable is associated with exactly one value of the dependent variable
- for every value in the domain there is a unique value in the range

function notation

- a symbolic notation used for writing a function
- $f(x)$ is read as "f of x" or "f at x".

Did You Know?

The term *function* was developed and used by German mathematician Gottfried Wilhelm Leibniz (1646–1716). The notation we use today was created by Swiss mathematician Leonhard Euler (1707–1783). Euler contributed to many branches of mathematics.

Functions can be written using **function notation**. The function $y = 4x + 1$ is written as $f(x) = 4x + 1$. The name of the function is f, with a variable name of x. In this example, $4x + 1$ is the rule that assigns a unique value for y for each value of x. Any letter may be used to name a function. Two examples are $v(t) = 9.8t^2$ and $A(r) = \pi r^2$.

Function notation highlights the input/output aspect of a function. The function $f(x) = 4x + 1$ takes any input value for x, multiplies it by 4, and adds 1 to give the result. For example, if $x = 2$ is the input, then $f(2) = 9$ is the output.

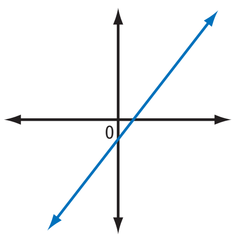
$f(2) = 4(2) + 1$
$f(2) = 9$

When $x = 2$, the value of the function is 9. The point (2, 9) is on the graph of the function.

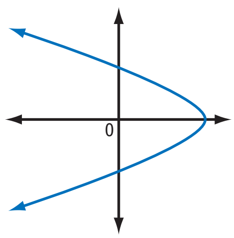
Example 1 Determine Whether a Relation Is a Function

For each pair of relations, decide which relation is a function and which is not a function. Explain your choice.

a) A



B



b) C

x	y
2	5
2	7
4	9
6	11

D

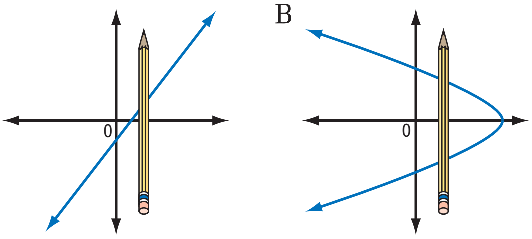
x	y
–3	3
–2	4
–1	3
0	4

c) E {(1, 1), (2, 2), (3, 3), (4, 4)}
F {(1, 1), (1, 2), (1, 3), (1, 4)}

Solution

For each pair, check to see which relation has a domain value associated with more than one range value. These are not functions.

a) When a relation is given as a graph, look for any points on the graph that line up vertically. If points line up vertically, a value of x has more than one corresponding value of y. One method to check this is called the **vertical line test**.



Relation A is a function, because it passes the vertical line test. Each domain value is used only once. Relation B does not pass the vertical line test. Many values of x have two values of y.

b) In relation C, the value of the relation when x equals 2 is associated with y-values of 5 and 7, so this is not a function. Relation D is a function, because each domain value is used only once.

c) In relation E, each domain value is used only once, so it is a function. In relation F, the domain value 1 is used repeatedly. Therefore, relation F is not a function.

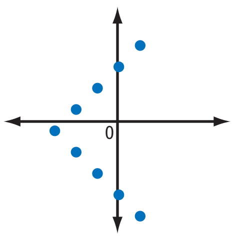
vertical line test

- a test to see if a graph represents a function
- if any vertical line intersects the graph at more than one point, the relation is not a function

Your Turn

Which of the following relations are functions? Explain your choices.

a)

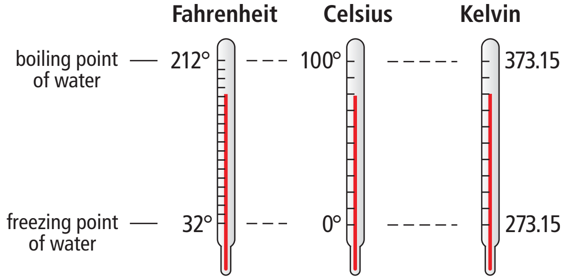


b) {(−2, 1), (0, 0), (2, 1), (5, 1)}

c)

x	y
1	3
2	3
3	4
4	4
5	4

Example 2 Work With Function Notation



The function $F(C) = 1.8C + 32$ is used to convert a temperature in degrees Celsius (°C) to a temperature in degrees Fahrenheit (°F).

a) Determine $F(25)$. Explain your answer.

b) Determine C so that $F(C) = 100$. Explain your answer.

Solution

a)

$$\begin{aligned} F(C) &= 1.8C + 32 \\ F(25) &= 1.8(25) + 32 \\ F(25) &= 45 + 32 \\ F(25) &= 77 \end{aligned}$$

This means that 25 °C is the same as 77 °F.

b)

$$\begin{aligned} F(C) &= 1.8C + 32 \\ 100 &= 1.8C + 32 \\ 100 - 32 &= 1.8C + 32 - 32 \\ 68 &= 1.8C \\ \frac{68}{1.8} &= \frac{1.8C}{1.8} \\ 37.8 &= C \end{aligned}$$

This means that 100 °F is the same as 37.8 °C.

Your Turn

a) Determine $F(86)$. Explain your answer.

b) Determine C so that $F(C) = 98.6$. Explain your answer.

c) Another measurement scale for temperature that is used in science is the Kelvin scale. The function $K(C) = C + 273.15$ can be used to convert from degrees Celsius to kelvins. Determine $K(80)$ and explain your answer.

Did You Know?

The Kelvin temperature scale was introduced by William Thomson, 1st Baron Kelvin (1824–1907), a Scottish physicist. Units on the scale are called kelvins. The Kelvin scale does not use the word *degrees* or the symbol °.

Example 3 Graphing Linear Functions

Skye has a cell phone plan for a monthly fee of \$20 plus 15¢ for each text message to or from a number not on a list of favourites. Skye's monthly bill can be modelled by the relation $C = 0.15n + 20$, where C is the total charge, in dollars, and n is the number of additional text messages.

a) Write the relation in function notation.

b) Make a table of values. Graph the function if Skye sends up to four additional text messages.

c) If Skye's cell phone bill for a certain month is \$22.25, how many additional text message charges are there?

Solution

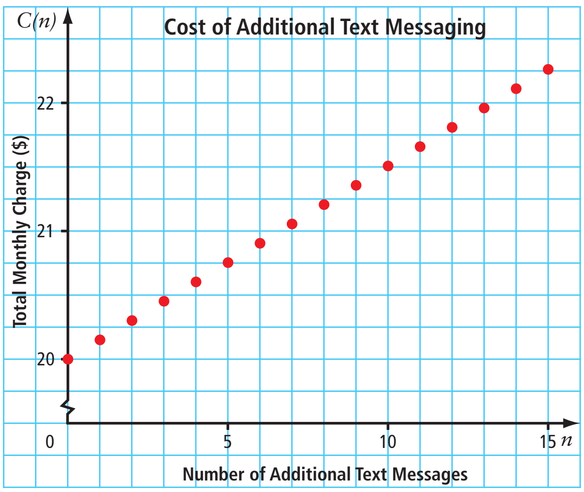
a) The independent variable is n, the number of additional text messages. The function is $C(n) = 0.15n + 20$.

b) Choose values for the independent variable, n. Use the formula to determine the total charge.

A table of values for the first five values of this function is shown.

n	$C(n)$
0	20.00
1	20.15
2	20.30
3	20.45
4	20.60

Continuing the pattern, the graph shows the first 16 values for the function.



Why are the points in this graph not connected?

c) Method 1: Use the Graph

Use the graph on page 309. Locate the value of \$22.25 on the total monthly charge axis. Read the corresponding number of text messages. From the graph, it can be seen that Skye's cell phone bill includes 15 additional text message charges.

Method 2: Use the Formula

The total charge is \$22.25.

Substitute $C = 22.25$ into the relation.

$$\begin{aligned} C(n) &= 0.15n + 20 \\ 22.25 &= 0.15n + 20 \\ 22.25 - 20 &= 0.15n + 20 - 20 \\ 2.25 &= 0.15n \\ \frac{2.25}{0.15} &= \frac{0.15n}{0.15} \\ 15 &= n \end{aligned}$$

Solve for *n*.

Skye's phone bill includes 15 additional text message charges.

Your Turn

Use the relation $y = 3x - 1$.

a) Write the relation in function notation using f for the name of the function.

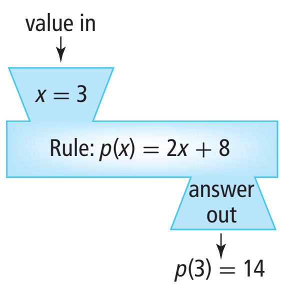
b) Make a table of values. Graph the function.

c) Determine the value of x if $f(x) = 53$.

Key Ideas

- All functions are relations but not all relations are functions.
- A relation is classified as a function if each value in the domain corresponds to exactly one value in the range.
- Each function has its own formula, or rule, that is often given using a special notation, called function notation.

  For example, $p(x) = 2x + 8$ shows that the function p takes an input value, multiplies it by 2, adds 8, and outputs the answer.



Check Your Understanding

Practise

1. Determine whether each relation is a function or is not a function. Give a reason for your answer.

a) (−1, 2), (0, 1), (1, 2), (2, 5)

b) (3, 12), (4, 12), (5, 14), (6, 14)

c) (1, 2), (2, 3), (3, 4), (4, 5), (5, 6)

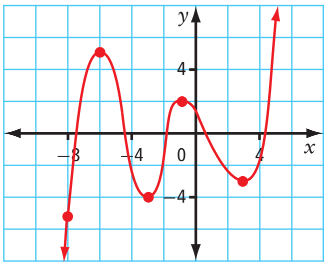
d)

x	y
0	0
1	−1
1	1
4	−2
4	2

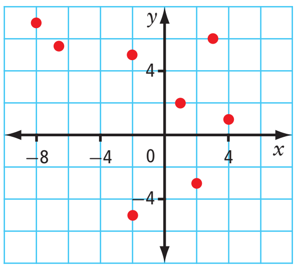
e)

Name	Age
Naomi	14
Wam	15
Brigid	14
Sharon	16
Arvind	15

f)



g)



2. The formula for the surface area, A, of a sphere with radius r is $A = 4\pi r^2$. Write this formula using function notation.

3. The cost to have artwork printed on T-shirts is given by the function $C(n) = 3n + 50$, where n is the number of shirts and C is the cost, in dollars. Write this function as a formula in two variables.

4. If $f(x) = 10x - 8$, determine

a) $f(2)$ **b)** $f(-3)$ **c)** x if $f(x) = 42$

5. If $h(x) = \frac{2}{3}x + 1$, determine

a) $h(9)$ **b)** $h(-3)$ **c)** x if $h(x) = -7$

6. Consider the function $p(x) = -4x + 2$.

a) What is the value of $p(0)$?

b) Determine x so that $p(x) = -2$.

7. Make a table of values and graph each function.

a) $g(x) = -3x + 5$ for the domain $\{-3, -2, -1, 0, 1, 2, 3\}$

b) $h(x) = \frac{x}{2}$ for the domain $\{x \mid -10 \leq x \leq 10, x \in \mathrm{R}\}$

Apply

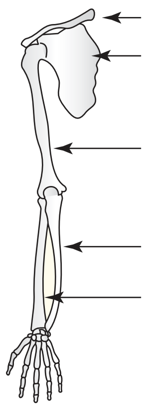
8. Mike currently has \$200 and saves \$20 each week. The function $M(w) = 20w + 200$ describes his saving pattern. Ali currently has \$200 and spends \$20 each week. The function $A(w) = 200 - 20w$ describes her spending pattern.

a) What does the variable w represent in each function?

b) Explain the meaning of $M(w)$ and $A(w)$.

c) What is the value of each function when $w = 4$? Explain your answer.

d) Determine the value of w when $A(w) = 0$. Explain your answer.



9. **Unit Project** Using skeletal remains, a forensic anthropologist can accurately determine the sex, race, age, and height of a person.

a) The height, h, in centimetres, of a male can be determined using the function $h(L) = 2.9L + 70.6$, where L is the length, in centimetres, of his humerus. Suppose you find a humerus of a male and measure the bone to be 36.87 cm in length. How tall was the man?

b) The function $h(L) = 2.8L + 71.4$ can be used to determine the height, h, in centimetres, of a female, where L is the length, in centimetres, of her humerus. Determine $h(36.87)$. What does $h(36.87)$ represent?

c) Height, h, in centimetres, can also be determined using the functions $h(L) = 3.3L + 86.4$ for a male and $h(L) = 3.3L + 81.3$ for a female. In these functions, L represents the length of the radius bone, in centimetres. What is an appropriate range for each of these functions? Explain.

d) Based on the range you determine in part c), what is the span of values (the domain) for the radius bone in males and in females?

e) Measure the length of your radius bone. Use the appropriate function to determine your height. How accurate is the prediction?

10. Weight on the moon is not the same as it is on Earth because of differences in the force of gravity. The function $m(E) = \frac{E}{6}$ can be used to approximate your weight, m, on the moon, where E represents your weight on Earth.

a) Does the function indicate that you would be heavier or lighter on the moon than on Earth? Explain.

b) If a person weighs 80 kg on Earth, how much would the person weigh on the moon?

c) How much would you weigh on the moon?

d) What is an appropriate domain for this function? Using this domain, make a table of values and graph the function.

11. In air, the speed of sound depends on the air temperature. The speed of sound, S, in kilometres per hour, can be estimated using the function $S(t) = 1192 + 2.2t$, where t is the temperature of the air, in degrees Celsius.

a) What is the speed of sound in air at a temperature of 20 °C?

b) At an altitude of 10 km, the air temperature is about −55 °C. What is the speed of sound at this altitude?

c) A jet "breaks the sound barrier" when it flies faster than the speed of sound. The aircraft's speed is then typically referred to in Mach numbers. A jet's Mach number can be determined using the function $M(V) = \frac{V}{S}$, where V is the speed of the jet and S is the speed of sound, both in kilometres per hour. Use your results for S from parts a) and b) to write the Mach number function when the temperature is 20 °C, and when the temperature is −55 °C.

d) A jet is flying at 1800 km/h at an altitude of 10 km. What is the jet's Mach number?

Water vapour created as jet breaks sound barrier

Extend

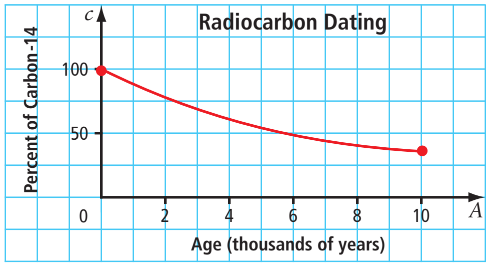
12. a) Sharon creates a function in the form $f(x) = ■x + ■$ for her classmates to solve. To figure out the actual equation, students give Sharon input values. She gives them the output from the function. The values are $f(1) = 5$, $f(2) = 8$, $f(-1) = -1$, and $f(-2) = -4$. What is the equation for Sharon's function?

b) Create a function of your own. Have someone determine the equation of your function by giving you input values and studying the pattern in your responses.

Did You Know?

First Nations hunters used a variety of approaches to hunt bison. Of all the methods the hunters devised, the most effective was the bison jump. At Head-Smashed-In Buffalo Jump, the cliff is about 10 m high. The oldest bones and stone tools at the jump are buried 10 m below this apron. This indicates that the cliff at one time was twice its current height.

13. **Unit Project** After an animal dies, the amount of radioactive carbon-14 in its bones declines. Archaeologists use this fact to determine the age of a bone based on the percent of carbon-14 remaining in the fossilized bones. The relation shows the age, A, in years, of an animal based on the percent, c, of carbon-14 remaining.



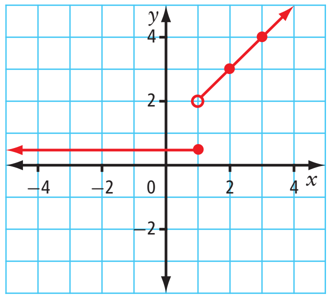
a) Is this relation a function? Why?

b) At Head-Smashed-In Buffalo Jump, in southwestern Alberta, the most recent bison bones found had 98% of the carbon-14 still remaining. From the graph, estimate the age of these bones.

c) The oldest bison bones found at Head-Smashed-In Buffalo Jump were about 5800 years old. Estimate the percent of carbon-14 still remaining in these bones.

14. **a)** Does the graph represent a function? Explain.

b) What is the value of $f(-4)$? $f(1)$? $f(3)$? $f(5)$?



15. The input for a function can be another function. If $h(x) = 2x - 5$, determine a simplified expression for each of the following.

a) $h(4x)$ **b)** $h(2x + 3)$ **c)** $h\left(\frac{x}{2} - 1\right)$

Create Connections

16. Explain, using examples, why some relations are not functions but all functions are relations. Draw a Venn diagram to illustrate the relationship between functions and relations.

17. Explain the difference between $f(2)$ and $f(x) = 2$.

18. Jean-Marie has never seen function notation. When he sees a question that asks him to determine the value of $f(x + 2)$, he gives his answer as $fx + 2f$.

a) How does Jean-Marie interpret the question?

b) Explain the meaning of this question to Jean-Marie in the context of functions.