## 6.1 Graphs of Relations

Focus on ...

- describing a possible situation for a graph
- sketching a graph for a given situation


Graphs are often used to visually represent the relationship between two or more things. The graphs shown are derived from the career statistics for two NHL goalies, Martin Brodeur and Roberto Luongo. One graph shows the number of goals scored against each goalie. The other shows the number of shots each faced. If you compare the graphs, what conclusions might you make?


## Materials

- ruler
- grid paper



## Investigate Describing and Sketching Graphs

1. a) Work in pairs. The graph shows the distance a rock climber is from the base of a cliff as time passes. Using the words climbing, resting, or descending, describe what the climber is doing during each segment shown. Explain your choice.

b) Is there more than one interpretation of the climber's actions during the times indicated by segments AB, CD, DE, and FG?
c) For any section that you listed as "climbing," how would you change the graph to show that the person is climbing faster? Explain your reasoning.
d) What would you add to the graph to show the climber's return to the bottom of the cliff?
2. Work in pairs. Match each graph with a situation from the list. Explain your choice. Suggest titles for each axis to show the quantities being compared.

a) the temperature of a cup of hot chocolate over time
b) a car accelerating to a constant speed
c) the distance a person walks during a hike
d) the height of a soccer ball kicked across a field
3. Work in small groups. Create a speed-time graph for the following scenario. Put speed on the vertical axis and time on the horizontal axis. Clearly describe each section of your graph. Then, pass your work to another group. Connor is riding his skateboard along a path. Almost immediately after leaving home, Connor travels down a short steep hill. At the bottom, the path makes a turn. The remainder of the trip is on relatively flat land. Connor kicks to keep moving. He then stops before a railway crossing. He also practises a few tricks along the way. He completes a basic "ollie" and performs a second ollie over a speed bump. Finally, after travelling at a constant rate for the last part of the trip, Connor arrives at his destination.
4. a) Review and discuss another group's graph.
b) How is it similar to yours? How is it different from yours?
5. Reflect and Respond How might each situation be shown on a graph?
a) one quantity is changing at a constant rate in relation to the other quantity
b) the rate of change is constant and the change is happening quickly
c) one quantity is not changing
d) a change in one quantity is not constant


## Link the Ideas

A graph is an effective way to show the relationship between two quantities. A constant rate of change is represented graphically by a straight line. The steepness of the line indicates the rate at which one quantity is changing in relation to the other.

A steeper line indicates a faster rate of vertical change on the red line than on the blue line. This change may indicate an increase or a decrease.


A horizontal line means that there is no rate of change.


Every value on the horizontal axis is related to the same value on the vertical axis.

Not all relationships are represented by a straight line. A curve shows that the rate of change is not constant.


As quantity $B$ increases, the increase in quantity $A$ is gradual at first. It then becomes much greater.


As quantity $B$ increases, the increase in quantity A slows until quantity A reaches a maximum value. Then, quantity A decreases.

## Example 1 Interpret a Graph

Wakeboarding has grown to be a popular water sport. The graph shows the distance that a wakeboarder is from her starting point on Last Mountain Lake in Saskatchewan. Describe what the boarder is doing.


## Solution



AB: Since the distance is increasing, the wakeboard rider is moving away from her

Why does the distance increase starting point. The change in distance starts slowly at first? slowly at first. It then reaches a constant rate.

BC: Since the distance is not changing, the rider has either stopped or is on a path that

What kind of path would allow this? keeps her at a constant distance from the starting point.

CD: The change in distance increases so that the wakeboard rider is moving away from her starting point at a quicker rate.
DE : Since the distance is decreasing quickly, the rider is moving toward the starting point at a fast rate.

EF: Since the distance is not changing, the rider has either stopped or is on a path that keeps her at a constant distance from the starting point.

FG: The distance is decreasing to zero. The rider is returning to the starting point at a constant rate.

How do you know the rate is constant?

## Your Turn

The graph shows the speed of the boat that is pulling a wakeboarder. Describe what the boat is doing.


## Example 2 Interpret a Graph



Which graph best represents bacteria growth if the bacteria's food supply is limited? Explain your choice.




## Solution

If the food supply is limited, the bacteria eventually will run out of food and die off. Graph A can be ruled out since it indicates continued growth.

Graph B is also not the correct choice. It shows the number of bacteria decreasing at the start while the food supply is high, reaching a low point, and then increasing.

Graph C is the correct choice. The increase in bacteria is initially slow but then goes through a period of rapid growth. The number remains stable for a while. Then the bacteria die off because there is no more food.

## Your Turn

Which graph best represents a person's height as the person ages? Explain your choice.





## Example 3 Graph a Situation

Josaphee leaves her home and walks to the store. After buying a drink, she slowly jogs to her friend's house. Josaphee visits with her friend for a while and then runs directly home. Using the distances shown, draw a distance-time graph that shows Josaphee's distance from her house. Explain each section of your graph.


## Solution

There is no change in Josaphee's distance from home
while she is visiting her friend. This line segment is

longer than the segment when Josaphee was at the store | To make a |
| :--- |
| tosaphee's distance |

## Your Turn

For the same scenario and using the distances shown, draw a distance-time graph that shows Josaphee's distance from the store.
Explain each section of your graph.

## Key Ideas

- When comparing two quantities, straight lines are used to indicate a constant change in the relationship. Curves are used when the rate of change is not constant. Horizontal lines are used if one quantity is not changing relative to a change in the other quantity.



## Check Your Understanding

## Practise

1. The graph shows how quantity $B$ is changing relative to quantity A. Describe each section of the graph as representing a constant increase, a constant decrease, an increase that is not constant, a decrease that is not constant, or no
 change. Explain your answers.
2. a) Match each scenario with its appropriate graph.
i) the speed of a train as it arrives at a station
ii) a football's distance above ground as the ball is kicked iii) the number of un-popped kernels as a popcorn maker heats up and pops the corn

b) Describe a scenario for the graph that you do not use in part a).
3. Sketch a copy of each graph. Label the axes using the choices given.

| Choices for <br> Vertical Axis | Choices for <br> Horizontal Axis |
| :--- | :--- |
| Profit From Sales | Time |
| Speed of ATV | Ticket Price |
| Height of Grass | Distance Travelled |

a)

b)

c)

4. Describe a possible scenario for each graph. Tell what each axis represents in each case.
a)

b)

c)


## Apply

5. Paul boards Vancouver's SkyTrain at the Main Street station. He travels east on the Millennium line toward the Columbia station. He falls asleep around the 29th Avenue station and does not wake up until the Rupert station. Paul decides to stay on the train until Commercial Drive, where he transfers to another eastbound train. He takes this train to the Columbia station. Make a distance-time graph for Paul's journey. Sketch his distance from the Columbia station versus time.

6. Formats for distributing recorded music have changed through the years. Study the multi-line graph. Predict which line represents each format:
 vinyl albums, cassette tapes, compact discs, and digital downloads. Explain your choices.
7. Uriash enjoys snowmobiling. The two graphs give information about one of his rides. Use them to describe what Uriash did.

8. The table gives the approximate amount of water needed for various activities. Sketch a graph showing your water usage from the moment you wake up until you go to bed. On the vertical axis, record the amount of water that you use. Record time on the horizontal axis. Include a description. needed for wh show

## Did You Know?

The first SkyTrain line, the Expo Line, was built in time for the Expo 86 World's Fair. It began operation in December 1985. The trains are made by a Canadian company, Bombardier.


| Water Use | Amount |
| :--- | :--- |
| Toilet flush | 6 L |
| Shower | $10 \mathrm{~L} / \mathrm{min}$ |
| Bath | 68 L |
| Sink faucet | $10 \mathrm{~L} / \mathrm{min}$ |
| Dishwasher | $27 \mathrm{~L} / l o a d$ |
| Washing machine | $99 \mathrm{~L} /$ load |

## WWW Web Link

To view a video that shows how the Coaster was built or to view a video of a ride on the roller coaster, go to www.mhrmath10.ca and follow the links.
9. The Coaster is a wooden roller coaster built in 1958 for the Pacific National Exhibition in Vancouver. Imagine yourself taking a ride. Follow all the ups, downs, twists, and turns. The letters on the track will help you follow the Coaster's path.

a) Sketch a height-time graph showing your height above the ground versus time for one complete ride.
b) Sketch a speed-time graph showing your speed versus time, for one complete ride.
10. After arriving home from work, Cari leaves to pick up her daughter Allie from daycare. She walks to the daycare and then walks home with her daughter. One hour later, Cari and Allie leave home by car to pick up Cari's other children. They go first to Jaime's school and then to Mathias's school. The diagram shows the distances to each location and Cari's route, marked in red.


Sketch a distance-time graph of each scenario. Starting from the moment Cari leaves for the daycare, show
a) Cari's distance from her home
b) Allie's distance from home
c) Jaime's distance from home
d) Mathias's distance from home
11. Create a speed-time graph for this scenario. A skydiver jumps from an airplane that is flying at a speed of $160 \mathrm{~km} / \mathrm{h}$. In about the first 10 s , the skydiver accelerates to a falling speed of $190 \mathrm{~km} / \mathrm{h}$. He stays at this speed because he has adopted the standard flat and stable, or "face to Earth," position. After another 30 s, the skydiver opens his
 parachute and quickly slows his descent to about $18 \mathrm{~km} / \mathrm{h}$. He maintains this speed until just before reaching the ground. Then he uses his parachute to slow down slightly, allowing him to make a smooth landing.

## Extend

12. Demography is the study of human populations. The graph shows changes in birth and mortality rates over time and their effects on the total population. Study the birth and mortality rates. Describe and explain the changes in the total population at each of the five stages marked.

13. a) Half-life is the time required for half of a sample of a radioactive substance to decay. The graph shows a typical decay curve for an isotope. What is the half-life of this radioactive substance?

b) Bismuth-210 has a half-life of 5 days. Make a graph showing a decay curve for this substance. Show the first 20 days of decay.
14. The graph shows fees to park a vehicle in a public parking lot in Calgary. Describe the rate scheme.


## Materials

- CBL interface with a motion detector
- computer or graphing calculator with appropriate software


## Create Connections

15. Explain why each graph represents an impossible situation.
a)

b)

16. Write a story less than one page in length that incorporates four general graphs: speed versus distance, temperature versus time, number of people versus time, and money versus number of people.
17. $\bar{M} \mid \bar{N} \bar{I} \bar{L} \bar{A} \bar{A} \bar{B}$ Use a motion detector connected to a computer or graphing calculator to collect and graph information about movement in your classroom.
Step 1 Load the data-collection program onto your computer or calculator.
Step 2 Connect the CBL interface to the calculator or computer.
Step 3 Connect the motion detector to any of the sonic ports on the CBL interface.

Step 4 Attach the motion detector to a desk or table. Make sure there is a clear path in front of the detector. The device emits ultrasonic waves that fill a cone-shaped area about $15^{\circ}$ to $20^{\circ}$ off the centre line of the detector. Keep objects such as desks or chairs out of the cone, because the waves may detect them and record incorrect data.
Step 5 Start the data-collection program.
Step 6 The motion detector measures the time it takes for ultrasonic waves to travel to an object and return to the sensor. As a class, take turns walking in front of the sensor in a manner that produces each of the following graphs.


Step 7 Have a classmate walk in front of the sensor in any way he or she likes. Observe the motion carefully. Create a distance-time graph for the motion. Compare your graph to the one produced on the calculator or computer.
Step 8 Repeat step 7 for other members of the class.

